

# Risk endogeneity at the lender/investor-of-last-resort\*

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## Abstract

Can a central bank de-risk its balance sheet by doing *more*? We propose a novel risk measurement framework to empirically study the time-variation in central bank portfolio credit risks associated with monetary policy operations. The framework accommodates numerous bank and sovereign counterparties, fat tails, skewness, and time-varying dependence parameters. In an application to selected items from the consolidated Eurosystem's weekly balance sheet between 2009 and 2015, we find that unconventional monetary policy operations tended to generate beneficial risk spill-overs across monetary policy operations, causing overall risk to be concave in exposures. Some policy operations were 'self-financing' in net risk terms.

**Keywords:** Credit risk; risk measurement; central bank; lender-of-last-resort; score-driven model.

**JEL classification:** G21, C33.

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# 1 Introduction

For at least 150 years, going back to [Baring \(1797\)](#) and [Bagehot \(1873\)](#), central bankers have wondered to what extent they make rather than take their own balance sheet risks. Theoretically, the possibility of the central bank making ones own risk is uncontroversial. In the context of a pure illiquidity crisis without solvency concerns, for example, the simple announcement by a central bank to act as a lender-of-last-resort (LOLR) in line with Bagehot-inspired principles could shift the economy from a ‘bad’ to a ‘good’ equilibrium, causing all illiquidity-related credit risks to quickly disappear at virtually no cost or additional balance sheet risk; see e.g. [Diamond and Dybvig \(1983\)](#), [Allen and Gale \(2000\)](#), and [Rochet and Vives \(2004\)](#). Whether this possibility is empirically relevant, however, is currently unclear, as reality rarely resembles the typical textbook case. In addition, empirical studies of central bank portfolio credit risks are rare, primarily because the required data are often confidential, proprietary but expensive, or publicly available but hard to find.

During the euro area sovereign debt crisis between 2010 and 2012, severe liquidity squeezes and market malfunctions forced the Eurosystem – the European Central Bank (ECB) and its then 17 national central banks (NCBs) – to act as a LOLR to the entire financial system; see e.g. [ECB \(2014\)](#) and [de Andoain et al. \(2016\)](#). Large-scale central bank lending to banks ensured the proper functioning of the financial system and, with it, the transmission of monetary policy. Such lending occurred mainly via main refinancing operations (MROs), multiple long-term refinancing operations (LTROs) with maturities of up to one year, two very-long-term refinancing operations (VLTROs) with a three-year maturity, as well as targeted LTROs (TLTROs), all backed by repeated expansions of the set of eligible collateral. In addition, the Eurosystem also acted as an investor-of-last-resort (IOLR) in stressed markets. For example, it purchased sovereign bonds in illiquid secondary markets within its Securities Markets Programme (SMP) between 2010 and 2012, and committed to doing so again under certain circumstances within its Outright Monetary Transactions (OMT) program as announced in August 2012.

The Eurosystem’s actions as a large-scale lender- and investor-of-last-resort during the euro area sovereign debt crisis had a first-order impact on the size, composition, and, ultimately, the risk of its balance sheet. At the time, its policies raised substantial concerns about

the central bank taking excessive risks. Particular concern emerged about the materialization of credit risk and its effect on the central bank’s reputation, credibility, independence, and ultimately its ability to steer inflation towards its target of close to but below 2% over the medium term. The credit risk concerns were so pronounced at the time that some media reports referred to the ECB unflatteringly as the ECBB: Europe’s Central Bad Bank; see e.g. [Brendel and Pauly \(2011\)](#) and [Böhme \(2014\)](#). On the upside, the Eurosystem’s experience during the euro area sovereign debt crisis is arguably an ideal laboratory to study the impact of a central bank’s unconventional policies on the risks inherent in its balance sheet.

It is uncontroversial that lending freely in line with [Bagehot \(1873\)](#)-inspired principles<sup>1</sup> during a liquidity crisis as well as purchasing sovereign bonds during an existential sovereign debt crisis can increase the credit risk of a central bank’s balance sheet. How different lender- and investor-of-last-resort policies interact from a risk perspective, however, is currently less clear. Specifically, we ask: Can central bank liquidity provision or asset purchases during a liquidity crisis be ‘self-financing’ in net risk terms? This could happen if risk-taking in one part of the balance sheet (e.g., more asset purchases) de-risks other balance sheet positions (e.g., the collateralized lending portfolio) by a commensurate or even larger amount. How economically important were such risk spillovers across policy operations? Were the Eurosystem’s financial buffers at all times sufficiently high to match the portfolio tail risks? Did past operations differ in terms of impact per unit of risk? Finally, can *other* central banks’ policy announcements spill over and affect the Eurosystem’s risks?

The methodological part of this paper proposes a novel credit risk measurement framework which allows us to study the above questions. The framework is based on a tractable high-dimensional dependence (copula) function that can accommodate a large number of bank and sovereign counterparties. The model is state-of-the-art in that it allows us to capture extreme joint tail dependence (fat tails), time-varying volatility and correlation parameters, as well as a potential asymmetry in the correlation dynamics. Our framework combines elements of earlier models put forward in [Creal et al. \(2011\)](#), [Creal et al. \(2014\)](#), and [Lucas et al. \(2014, 2017\)](#), which are here modified to accommodate a large number of counterparties and asymmetric correlation dynamics.

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<sup>1</sup>[Bagehot \(1873\)](#) argued that any liquidity support should be against good collateral, only to solvent banks, and at penalty rates when compared to normal times.

The central bank’s risk management function is different from that of a commercial bank in at least three ways. First, unlike for commercial banks, risk and profitability are not first-order measures of success for a central bank. When taking monetary policy decisions the financial consequences for the central bank’s profit and loss statement are usually not a primary concern. If a central bank endures sustained losses, however, its independence may be, or perceived to be, impinged, which in turn may have consequences for its ability to achieve its goals. Further, central bank profits are almost always distributed to sovereign treasuries, thus contributing to the public budget. In this sense, central bank profits have fiscal consequences; see e.g. [Del Negro and Sims \(2015\)](#). Because of this, and increasingly since the financial crisis, central banks as public institutions face scrutiny over their activities and financial risks.

Second, commercial banks, by engaging in maturity transformation, are by their very nature exposed to liquidity shocks. Central banks are uniquely able to provide liquidity-support in a liquidity crisis owing to the fact that they are never liquidity-constrained in the currency they issue; see e.g. [Reis \(2015\)](#), and [Bindseil and Laeven \(2017\)](#). Consequently, the default risk of the central bank itself is zero at all times, at least regarding its liabilities in domestic currency.

Finally, a small or medium-sized commercial bank is unlikely to be able to materially influence financial risks and risk correlations associated with the bank-sovereign nexus. Commercial banks are ‘risk takers’ in more than one sense – risk management is primarily a function of expediently choosing exposures at given risks. This is inherently less true for central banks, as we show, particularly during a financial crisis.

The empirical part of this paper applies our high-dimensional credit risk framework to exposures associated with the Eurosystem’s major conventional and unconventional monetary policy operations. Exposures are taken from the Eurosystem’s balance sheet and measured at a weekly frequency between 2009 and 2015. Point-in-time risk measures are obtained from Moody’s Analytics (for banks) or are inferred from CDS spreads (for sovereigns), also at a weekly frequency. All risk model parameters are estimated by the method of maximum likelihood. Standard portfolio risk measures, such as the expected loss and expected shortfall, are subsequently obtained through Monte Carlo simulation. We compare the model-implied portfolio credit risks shortly before and after key policy announcements to study the time

differences associated with different monetary policy operations. A ‘high-frequency’ (weekly) assessment allows us to identify the effect of each policy on the relevant portfolio credit risks; see e.g. [Rogers et al. \(2014\)](#), [Fratzcher and Rieth \(2015\)](#), and [Krishnamurthy et al. \(2018\)](#) for similar event study approaches. To distinguish size from balance sheet composition effects we consider changes in portfolio credit risks both in absolute terms and in percentages of total assets.

We focus on four empirical findings. First, we find that LOLR- and IOLR-implied credit risks are usually negatively related in our sample. Taking risk in one part of the central bank’s balance sheet (e.g., the announcement of asset purchases within the SMP) tended to de-risk other positions (e.g., collateralized lending from previous LTROs). Vice versa, the allotment of two large-scale VLTRO credit operations each decreased the expected shortfall of the SMP asset portfolio. This negative relationship implies that central bank risks can be convex in exposures: In bad times, doubling size less than doubles risk. Conversely, reducing balance sheet size may not reduce risk by as much as one could expect by linear scaling. Risk spillovers between monetary policy operations are economically significant, and are similar in sign and magnitude around the time of the policy announcements.

Second, some unconventional policy operations were ‘self-financing’ in net risk terms. (I.e., not adding risk to the balance sheet in bottom line terms.) For example, we find that the initial OMT announcement de-risked the Eurosystem’s balance sheet by €−32.3 bn in 99% expected shortfall (ES). The announcement of OMT technical details in September 2012 was associated with a further reduction in 99% ES of €−21.0 bn. As another example, the allotment of the first VLTRO on 21 December 2011 raised the 99% ES associated with VLTRO lending from zero to approximately €30.6 bn. However, it also sharply reduced the need for shorter-term funding, and in addition de-risked the SMP asset portfolio (as banks invested some of the liquidity in government bonds). As a result, the overall 99% ES *decreased* by €1.1 bn. We conclude that, in extreme situations, a central bank can de-risk its balance sheet by doing more, confirming Bagehot’s well-known dictum that occasionally “only the brave plan is the safe plan.”

Such reductions in net risk are by no means guaranteed, however. The initial announcement of the SMP on 10 May 2010, for example, and the purchases that were implemented in that week, raised the 99% ES of SMP asset holdings from zero to approximately €7.8 bn.

The announcement and initial purchases also de-risked the collateralized lending book, but only mildly so. As a result, the overall 99% ES increased by €5.1 bn.

Third, our risk estimates allow us to rank past unconventional monetary policies in terms of their ex-post ‘risk efficiency’. Risk efficiency is the notion that a certain amount of expected policy impact should be achieved with a minimum level of additional balance sheet risk. Put differently, policy impact should be maximal given a certain level of additional balance sheet risk. Given an estimate of policy impact (e.g., a change in long-term inflation swap rates around the time of a policy announcement) and an appropriate estimate of risk (e.g., a change in expected losses), it is possible to evaluate different policies ex-post by scaling the former by the latter. Doing so, we find that the OMT program dominates the SMP and VLTROs in terms of ex-post risk efficiency. The first allotment of VLTRO funds appears to have been more risk-efficient than its second installment. The SMP, despite its benefits documented elsewhere, does not appear to have been a particularly risk-efficient policy measure.

Finally, we ask to what extent policy announcements of *other* central banks have influenced the Eurosystem’s risks. For example, the Federal Reserve’s announcement to ‘taper off’ asset purchases on 22 May 2013, or the announcement by the Swiss National Bank to unpeg the Swiss Franc from the Euro on 15 January 2015, could in principle have had a pronounced impact on the Eurosystem’s portfolio credit risks via an impact on the euro area financial sector or its risk correlations with sovereigns. We do not find this to be the case as expected losses barely move. International risk spillovers are negligible for four selected cases.

Our findings can have important implications for the design of central banks’ post-crisis operational frameworks. In addition, they can inform a debate on how to balance the need for a lender/investor-of-last-resort during liquidity crises with recent banking-sector regulations that seek to lower the frequency of such crises. As one key takeaway, a certain amount of excess liquidity for monetary policy purposes can be achieved via both credit operations and asset purchases. We find that collateralized credit operations imply substantially less credit risks (by a factor of approximately 1/40 to 1/70 in our crisis sample) than outright sovereign bond holdings per €1 bn of liquidity. Implementing monetary policy via credit operations rather than asset holdings, whenever possible, appears preferable from a risk

efficiency perspective. Second, expanding the set of eligible assets during a liquidity crisis can help mitigate the procyclicality inherent in most central bank’s risk protection frameworks. Our results suggest that doing so does not automatically increase the central bank’s credit risks, particularly if the relevant haircuts are set in an appropriate way.

The remainder of the paper is set up as follows. Subsection 1.1 discusses the related literature. Section 2 presents our exposure and risk data. Section 3 introduces our high-dimensional credit risk measurement framework. Section 4 applies the framework to a subset of the Eurosystem’s balance sheet. Section 5 concludes. A Web Appendix presents additional results and technical details.

## 1.1 Related literature

Our study relates to at least four directions of current research. First, several studies investigate the central bank’s role of LOLR and IOLR during a liquidity crisis. Important contributions include [Bagehot \(1873\)](#), [Diamond and Dybvig \(1983\)](#), [Allen and Gale \(2000\)](#), and [Rochet and Vives \(2004\)](#). [Freixas et al. \(2004\)](#) provide a survey; see also [Bindseil \(2014\)](#) for a textbook treatment.

Second, a nascent strand of literature applies stress-testing methods to central banks’ assets and income. [Carpenter et al. \(2013\)](#) and [Greenlaw et al. \(2013\)](#) stress-test the Federal Reserve’s ability to send positive remittances to the U.S. Treasury given that a large-scale sovereign bond portfolio exposes the Fed (and thus indirectly the Treasury) to interest rate risk. [Christensen et al. \(2015\)](#) advocate the use of probability-based stress tests, and find that the risk of suspended Fed remittances to the Treasury is small but non-negligible (at about 8%). Finally, [Del Negro and Sims \(2015\)](#) consider conditions under which a central bank might need to withhold seigniorage, or request recapitalization from the treasury, in order to maintain its monetary policy commitments.

Third, we effectively apply ‘market risk’ methods to solve a ‘credit risk’ problem. As a result, we connect a growing literature on non-Gaussian volatility and dependence modeling with another growing literature on portfolio credit risk and loan loss simulation. Time-varying parameter models for volatility and dependence have been considered, for example, by [Engle \(2002\)](#), [Demarta and McNeil \(2005\)](#), [Creal et al. \(2011\)](#), [Zhang et al. \(2011\)](#),

and Engle and Kelly (2012). At the same time, credit risk models and portfolio tail risk measures have been studied, for example, by Vasicek (1987), Lucas et al. (2001), Lucas et al. (2003), Gordy (2000), Gordy (2003), Koopman et al. (2011), Koopman et al. (2012), and Giesecke et al. (2014). We argue that our combined framework yields the best of these two worlds: portfolio credit risk measures (at, say, a one-year-ahead horizon) that are available at a market risk frequency (such as daily or weekly) for portfolio credit risk monitoring and impact assessments in real time.

Finally, to introduce time-variation into our empirical model specification we endow our model with observation-driven dynamics based on the score of the conditional predictive log-density. Score-driven time-varying parameter models are an active area of recent research, see for example Creal et al. (2011), Creal et al. (2013), Harvey (2013), Creal et al. (2014), Harvey and Luati (2014), Massacci (2016), Oh and Patton (2018), Lucas et al. (2018), and many more.<sup>2</sup> For an information theoretical motivation for the use of score-driven models, see Blasques et al. (2015), and for a forecasting perspective Koopman et al. (2016).

## 2 Eurosystem operations and data

We are interested in studying the time variation in Eurosystem portfolio credit risks, with a particular focus on such risks just before and after monetary policy announcements. We focus on six key announcements that are related to three unconventional monetary policy operations during the euro area sovereign debt crisis: the SMP, the VLTROs, and the OMT. This section first discusses these operations, and subsequently presents the relevant point-in-time risk data.

### 2.1 (Un)conventional monetary policy operations

The Eurosystem adjusts the money supply in the euro area mainly via so-called refinancing operations. Eurosystem refinancing operations between 2009 and 2015 included main refinancing operations (MROs), long-term refinancing operations (LTROs), very-long-term refinancing operations (VLTROs), and targeted long-term refinancing operations (TLTROs).

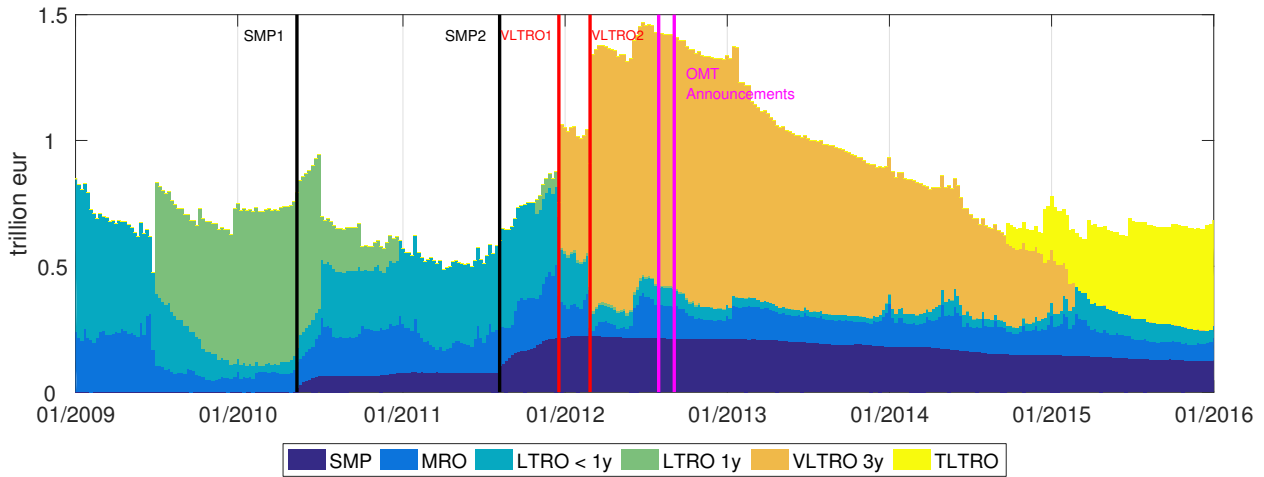
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<sup>2</sup>We refer to <http://www.gasmodel.com> for an extensive enumeration of recent work in this area.



Figure 1: Eurosystem collateralized lending and SMP exposures

Total collateralized lending exposures associated with different liquidity operations (MRO, LTRO<1y, LTRO1y, VLTRO3y, TLTRO), as well as sovereign bond holdings from purchases within the Securities Markets Programme (SMP). Vertical axis is in trillion euro. Data is weekly between 2009 and 2015. The SMP1 horizontal line refers to the initial announcement of the SMP on 10 May 2010. SMP2 marks the cross-sectional extension of the program on 08 August 2011. VLTRO1 marks the allotment of the first three-year VLTRO on 20 December 2011. VLTRO2 marks the allotment of the second three-year VLTRO on 28 February 2012. OMT1 marks the initial announcement of the OMT on 02 August 2012. OMT2 marks the announcement of the OMT’s technical details on 06 September 2012.



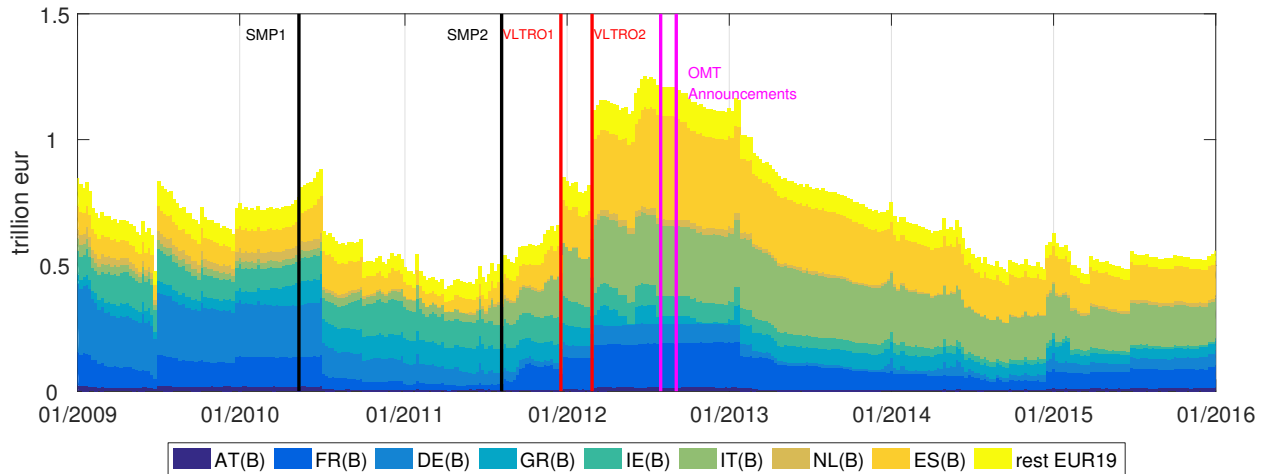
Before the onset of the global financial crisis in 2007, MROs and three-month LTROs were sufficient to steer short-term interest rates, to manage aggregate liquidity, and to signal the monetary policy stance in the euro area. Following the onset of the global financial crisis, however, the Eurosystem was forced to significantly extend the scale and maturity of its operations to include one-year LTROs and three-year VLTROs. TLTROs were set up in June 2014 mainly to further support (subsidize) bank lending to the non-financial sector. Between 2010 and 2012 the Eurosystem also conducted asset purchases within its SMP program.

Figure 1 plots selected items of the Eurosystem’s weekly balance sheet between 2009 and 2015.<sup>3</sup> We distinguish five different liquidity operations: MRO, LTRO<1y, LTRO1y, VLTRO3y, and TLTRO. The figure also plots the par value of assets held in the SMP portfolio. Clearly, the Eurosystem’s balance sheet varied in size, composition, and thus

<sup>3</sup>The Eurosystem’s weekly balance sheet is public; see <http://sdw.ecb.europa.eu/browse.do?node=9691110>.

Figure 2: Eurosystem collateralized lending across countries

Exposures across different euro area countries from five liquidity-providing operations; see Figure 1. The vertical axis is in trillion euro. Vertical lines indicate the events described in Figure 1. Data is weekly between 2009 and 2015.



credit riskiness during the course of the global financial crisis and euro area sovereign debt crisis. A peak in total assets was reached at the height of the debt crisis in mid-2012, at approximately €1.5 trn, following two VLTROs and SMP government bond purchases.

Figure 2 plots the Eurosystem’s country-level collateralized lending exposures, aggregated over the five liquidity-providing operations of Figure 1. The largest share of VLTRO funds was tapped by banks in Italy and Spain, and also Greece, Ireland, and Portugal. These sovereigns (and their banks) were perceived by markets to be particularly affected by the euro area sovereign debt crisis. Banks from non-stressed countries such as Germany and France were less liquidity-constrained and therefore relied less heavily on Eurosystem funding during the crisis.

The remainder of this subsection briefly reviews the three major unconventional monetary policy operations in chronological order: the SMP, the VLTROs, and the OMT. Each of these had a substantial impact on asset prices, point-in-time credit risks, and time-varying risk correlations; see, e.g., [ECB \(2014\)](#) for a survey.

## The SMP

The SMP was announced on 10 May 2010, with the objective to help restoring the monetary policy transmission mechanism by addressing the malfunctioning of certain government bond markets. The SMP consisted of interventions in the form of outright purchases which were aimed at improving the functioning of these bond markets by providing “depth and liquidity;” see [González-Páramo \(2011\)](#). Implicit in the notion of market malfunctioning is the notion that government bond yields can be unjustifiably high and volatile. For example, market-malfunctioning can reflect the over-pricing of risk due to illiquidity as well as contagion across countries; see [Constâncio \(2011\)](#). SMP purchases were not intended to affect the money supply. For this reason the purchases were sterilized at the time.

SMP interventions occurred in government debt securities markets between 2010 and 2012 and initially focused on Greece, Ireland, and Portugal. The SMP was extended to include Spain and Italy on 08 August 2011. Approximately €214 billion (bn) of bonds were acquired between 2010 and early 2012; see [ECB \(2013a\)](#). The SMP’s weekly cross-country breakdown of the purchase data is confidential at the time of writing. However, the ECB released its total cross-country SMP portfolio holdings at the end of 2012 in its 2013 Annual Report. At the end of 2012, the Eurosystem held approximately €99.0bn in Italian sovereign bonds, €30.8bn in Greek debt, €43.7bn in Spanish debt, €21.6bn in Portuguese debt, and €13.6bn in Irish bonds; see [ECB \(2013a\)](#). For impact assessments of SMP purchases on bond yields, CDS spreads, and liquidity risk premia see e.g. [Eser and Schwaab \(2016\)](#), [Ghysels et al. \(2017\)](#), and [De Pooter et al. \(2018\)](#).

## The VLTROs

Two large-scale VLTROs were announced on 08 December 2011, and subsequently allotted to banks on 21 December 2011 and 29 February 2012. The first installment provided more than 500 banks with €489 bn at a low (1%) interest rate for the exceptionally long period of three years. The second installment in 2012 was even larger, and provided more than 800 euro area banks with €530 billion in three-year low-interest loans. By loading up on VLTRO funds stressed banks could make sure they had enough cash to pay off their own maturing debts, and at the same time keep operating and lending to the non-financial sector.

Incidentally, banks used some of the money to also load up on domestic government bonds, temporarily bringing down sovereign yields. This eased the debt crisis, but may also have affected the bank-sovereign nexus (risk dependence) at the time; see e.g. [Acharya and Steffen \(2015\)](#).

## The OMT

On 26 July 2012, the president of the ECB pledged to do whatever it takes to preserve the euro, and that “it will be enough.” The announcement of Outright Monetary Transactions (OMT), a new conditional asset purchase program, followed shortly afterwards on 02 August; see [ECB \(2012\)](#). The OMT technical details were announced on 06 September 2012. The details clarified that the OMT replaced the SMP, and that, within the OMT, the ECB could potentially undertake purchases (“outright transactions”) in secondary euro area sovereign bond markets provided certain conditions were met. OMT interventions were stipulated to be potentially limitless, to focus on short-maturity bonds, and to be conditional on the bond-issuing countries agreeing to and complying with certain domestic economic measures determined by euro area heads of state. In the years since its inception, the OMT never had to be used. Nevertheless, its announcement is widely credited for ending the acute phase of the sovereign debt crisis by restoring confidence; see e.g. [Wessel \(2013\)](#).

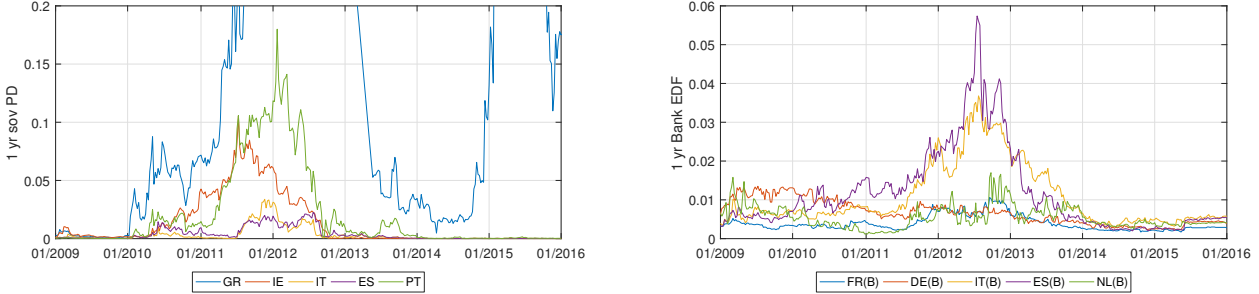
## 2.2 Bank and sovereign EDFs

We rely on expected default frequency (EDF) data from Moody’s Analytics, formerly Moody’s KMV, when assigning point-in-time probabilities of default (pds) to Eurosystem bank counterparties. EDFs are point-in-time forecasts of physical default hazard rates, and are based on a proprietary firm value model that takes firm equity values and balance sheet information as inputs; see [Crosbie and Bohn \(2003\)](#) for details. EDFs are standard credit risk measurements and are routinely used in the financial industry and credit risk literature; see for example [Lando \(2003\)](#), [Duffie et al. \(2007\)](#) and [Duffie et al. \(2009\)](#).

EDF measures are available for listed banks only. Many Eurosystem bank counterparties, however, are not listed. At the same time, some parsimony is required when considering many bank counterparties. We address both issues by using one-year-ahead median EDFs at the

Figure 3: Sovereign and banking sector EDFs

Right panel: CDS-implied-EDFs for five SMP sovereigns. Left panel: country-level banking sector EDF indices for ‘big-5’ euro area countries: Germany, France, Italy, Spain, the Netherlands. Data is weekly between 2009 and 2015.



country-level to measure point-in-time banking sector risk. EDF indices based on averages weighted by total bank assets are also available, but appear less reliable. The left panel of Figure 3 plots our EDF indices for the ‘big-5’ euro area countries: Germany, France, Italy, Spain, and the Netherlands. During the crisis, most Eurosystem liquidity was taken up by banks located in these countries; see Figure 2. Banking sector EDF measures differ widely across countries, and peak around mid-2012.

Unfortunately, firm-value based EDF measures are unavailable for sovereigns. We therefore need to infer physical probabilities of default from observed sovereign CDS spreads. Web Appendix A provides the details of our approach. To summarize, we first invert the CDS pricing formula of O’Kane (2008) to obtain risk-neutral default probabilities. We do this at each point in time for multiple CDS contracts at different maturities between 1 and 10 years. Second, we convert the risk-neutral probabilities into physical ones using the nonlinear mapping fitted by Heynderickx et al. (2016). Finally, we fit a Nelson Siegel curve to the term structure of CDS-implied-EDFs, and integrate the curve over the [0,1] year interval to obtain one-year-ahead CDS-implied-EDFs. The left panel of Figure 3 presents our sovereign risk measures for the five SMP countries.

### 3 Statistical model

#### 3.1 Portfolio risk measures

Credit losses at time  $t = 1, \dots, T$  over a one-year-ahead horizon are only known with certainty after the year has passed, and uncertain (random) at time  $t$ . The probability distribution of ex-ante credit losses is therefore a key concern for risk measurement. We model total credit losses  $\ell_t(k)$  associated with potentially many counterparties  $i = 1, \dots, N_t(k)$  as

$$\ell_t(k) = \sum_{i=1}^{N_t(k)} \ell_{it}(k) = \sum_{i=1}^{N_t(k)} \text{EAD}_{it}(k) \cdot \text{LGD}_{it} \cdot 1(\text{default}_{it}), \quad (1)$$

where  $k = 1, \dots, K$  denotes monetary policy operations (e.g., LTRO lending or SMP asset holdings),  $\ell_{it}(k)$  is the counterparty-specific one-year-ahead loss between week  $t+1$  and  $t+52$ ,  $\text{EAD}_{it}(k)$  is the exposure-at-default associated with counterparty  $i$  and policy operation  $k$ ,  $\text{LGD}_{it} \in [0, 1]$  is the loss-given-default as a fraction of  $\text{EAD}_{it}(k)$ , and  $1(\text{default}_{it})$  is an indicator function that takes the value of one if and only if counterparty  $i$  defaults. A default happens when the log-asset value of counterparty  $i$  falls below its default threshold; see e.g. [Merton \(1974\)](#) and [CreditMetrics \(2007\)](#). The loss  $\ell_{it}(k)$  is random because it is a function of three random terms,  $\text{EAD}_{it}$ ,  $\text{LGD}_{it}$ , and the default indicator. Total losses from monetary policy operations are given by  $\ell_t = \sum_{k=1}^K \ell_t(k)$ . We focus on the one-year-ahead horizon as it coincides with typical reporting frequencies.

Portfolio risk measures are typically based on moments or quantiles of the ex-ante loss distribution. We focus on standard risk measures such as the expected loss, value-at-risk at a given confidence level  $\gamma$ , and expected shortfall at confidence level  $\gamma$ . These risk measures are given respectively by

$$\begin{aligned} \text{EL}(k)_t &= \mathbb{E}[\ell_t(k)], \\ \Pr[\ell_t(k) \geq \text{VaR}^\gamma(\ell_t(k))] &= \gamma, \\ \text{ES}(k)_t^\gamma &= \mathbb{E}[\ell_t(k) \mid \ell_t(k) \geq \text{VaR}^\gamma(\ell_t(k))], \end{aligned}$$

where  $\mathbb{E}[\cdot]$  is the expectation over all sources of randomness in (1). The expected shortfall

$ES(k)_t^\gamma$  is often interpreted as the “average VaR in the tail,” and is typically more sensitive to the shape of the tail of the loss distribution. The subscript  $t$  indicates that the time series of portfolio risk measures is available at a higher than annual frequency (e.g., weekly).

The remainder of this section reviews the modeling of the ingredients of (1) from right to left: dependent defaults, LGD, and EAD.

### 3.2 Copula model for dependent defaults

Our model for dependent defaults follows closely from the frameworks developed in Lucas et al. (2014, 2017). To tailor the model to the problem at hand, however, we need to modify it to accommodate a large number of counterparties. In addition, owing to high dimensions, we seek to capture any potential asymmetry in the copula in a computationally straightforward and reliable way.

Following the seminal framework of Merton (1974) and CreditMetrics (2007), we assume that a counterparty  $i$  defaults if and only if its log asset value falls short of a certain default threshold. We assume that this happens when *changes* from current log asset values to future ones are sufficiently negative. Specifically, we assume that a default occurs with a time-varying default probability  $p_{it}$ , where

$$p_{it} = \Pr[\tilde{y}_{it} < \tau_{it}] = F(\tau_{it}) \Leftrightarrow \tau_{it} = F^{-1}(p_{it}), \quad (2)$$

where  $\tilde{y}_{it}$  is a one-year-ahead change in log asset value,  $\tau_{it}$  is a default threshold expressed as a log return, and  $F$  is the CDF of  $\tilde{y}_{it}$ . We stress that (2) is different from a typical Merton (1974) model in at least two ways. First, unlike Merton (1974),  $p_{it}$  is treated as an observed *input* in our model. Second,  $\tau_{it}$  does not have an economic interpretation in terms of debt levels of the firm. Rather,  $\tau_{it}$  is chosen at each point in time and for each counterparty such that the marginal default probability implied by the multivariate (copula) model coincides with the observed market-implied default probability for that counterparty at that time; see the last equality in (2). The reduced form character of (2) ensures that the model can be used for sovereigns as well, for which asset values are a less intuitive notion.

When modeling dependent defaults, we link default indicators using a Student’s  $t$  copula function. In particular, we assume that one-year-ahead changes in log-asset values  $\tilde{y}_{it}$  are

generated by a high-dimensional multivariate Student's  $t$  density

$$\tilde{y}_{it} = \mu_{it} + \sqrt{\zeta_t} L_{it}^{(k)} z_{it}, \quad i = 1, \dots, N_t(k), \quad (3)$$

where  $z_t^i \sim N(0, I_N)$  is a vector of standard normal risk terms,  $L_{it}^{(k)}$  is the  $i$ th row of  $L_t^{(k)}$ ,  $L_t^{(k)}$  is the Choleski factor of the Student's  $t$  covariance matrix  $\Omega_t^{(k)} = L_t^{(k)} L_t^{(k)'}$ ,  $\zeta_t \sim \text{IG}(\frac{\nu}{2}, \frac{\nu}{2})$  is an inverse-gamma distributed scalar mixing variable that generates the fat tails in the copula, and  $\nu$  is a degrees of freedom parameter that can be estimated. The covariance matrix  $\Omega_t^{(k)}$  depends on  $k$  because different counterparties participate in different monetary policy operations. We can fix  $\mu_{it} = 0$  in (3) without loss of generality since copula quantiles shift linearly with the mean.

Lucas et al. (2014) find that default dependence across euro area sovereigns is asymmetric, and well-captured by a (Generalized Hyperbolic) skewed- $t$  copula. Lucas et al. (2017) confirm this finding for banks. Rather than modeling any potential asymmetry in default dependence via (4), however, we introduce asymmetry in a novel way via the transition equation governing the correlation parameters in  $\Omega_t^{(k)}$  as detailed in the next subsection. This has the advantage that the quantiles of a standard  $t$ -density can be used in the estimation of the copula parameters. These quantiles are almost always tabulated and thus quickly available in standard software packages. By contrast, quantiles of skewed densities such as the Generalized Hyperbolic skewed- $t$  density used in Lucas et al. (2014) are not usually tabulated and need to be solved for numerically. Repeated numerical integration within a line search is time-consuming in high-dimensional applications, and in practice less reliable when  $\tau_{it}$  is far in the tail.

### 3.3 Score-driven copula dynamics

The time-varying covariance matrix  $\Omega_t^{(k)}$  in (4) is typically of a very high dimension. For example, more than 800 banks participated in the Eurosystem's second VLTRO program. The high dimensions and time-varying size of  $\Omega_t^{(k)}$  imply that it is difficult to model directly. We address this issue by working with block equi-correlations within and across countries. This approach specifies  $\Omega_t^{(k)}$  as a function of a much smaller covariance matrix  $\Sigma_t$  that is independent of  $k$ . We refer to e.g. Engle and Kelly (2012) and Lucas et al. (2017) for theory



and empirical applications based on dynamic (block-)equicorrelation matrices.

The smaller covariance matrix  $\Sigma_t$  is specified to depend on a vector of latent correlation factors  $f_t$ . Specifically,  $\Omega_t^{(k)} = \Omega_t^{(k)}(\Sigma_t(f_t))$ , where  $\Sigma_t(f_t) \in \mathbb{R}^{D \times D}$ , and  $D \ll N_t(k)$ . Our empirical application below considers nine banking sector risk indices and five SMP countries, thus  $D = 14 \ll N_t(k)$  at any  $t$  and  $k$ . The mapping of matrix elements  $\Omega_t^{(k)}(i, j) = \Sigma_t(l(i), m(j))$  is surjective but not injective. I.e., any element of  $\Sigma_t$  typically appears multiple times in  $\Omega_t^{(k)}$ . All bank correlation pairs across countries can be taken from  $\Sigma_t$ . The within-country correlation pairs – the off-diagonal elements in the diagonal blocks of  $\Omega_t^{(k)}$  – cannot be read off  $\Sigma_t$ . We therefore need to make an additional assumption and set the within-country bank correlations equal to the maximum (bank) row entry of  $\Sigma_t$ . As a result banks within each country are as correlated as the maximum estimated bank correlation pair across borders at that time. We expect this approach to yield conservative results given a pronounced degree of banking sector integration in the euro area, at least within non-stressed and stressed country blocks; see e.g. [ECB \(2013c\)](#) and [Lucas et al. \(2017\)](#).

Our approach for modeling  $\Sigma_t(f_t)$  builds on the approach of [Creal et al. \(2011\)](#). In this framework, the Choleski decomposition of  $\Sigma_t$  is specified in terms its polar coordinates and the factors  $f \in \mathbb{R}^{D(D-1)/2 \times 1}$  specify the relevant ‘angles’ of these coordinates. This setup ensures that  $\Sigma_t$  is always positive definite and symmetric.

The  $D$ -dimensional multivariate Student’s  $t$  density implied by the mixture (3) is given by

$$p(\tilde{y}_t; \Sigma_t, \nu) = \frac{\Gamma((\nu + N_t(k))/2)}{\Gamma(\nu/2)[(\nu - 2)\pi]^{N_t(k)/2} |\Sigma_t|^{1/2}} \cdot \left[ 1 + \frac{\tilde{y}_t' \Sigma_t^{-1} \tilde{y}_t}{(\nu - 2)} \right]^{-\frac{\nu + N_t(k)}{2}}, \quad (4)$$

where  $\Sigma_t$  is used instead of  $\Omega_t^{(k)}$ ,  $\Gamma(\cdot)$  is the gamma function, and  $\nu$  is the degrees of freedom parameter. The dynamics of  $f_t$  are specified by the transition equation

$$f_{t+1} = \omega \cdot \text{vech}(\rho(\bar{\Sigma})) + A \cdot S_t \nabla_t + B \cdot f_t, \quad (5)$$

where  $\omega$ ,  $A$ , and  $B$  are parameters and matrices to be estimated,  $\text{vech}(\rho(\bar{\Sigma})) \in \mathbb{R}^{(D-1)^2 \times 1}$  contains appropriately transformed unconditional correlations, the scaling matrix  $S_t$  is chosen as the inverse conditional Fisher information matrix  $E_{t-1}[\nabla_t \nabla_t']^{-1}$ , and  $\nabla_t$  is the score of

(4) given by

$$\begin{aligned}\nabla_t &= \frac{\partial \text{vech}(\Sigma_t(f_t))'}{\partial f_t} \cdot \frac{\partial \log p(y_t; \Sigma_t, \nu)}{\partial \text{vech}(\Sigma_t)} \\ &= \frac{1}{2} \Psi_t' \mathcal{D}'_D(\Sigma_t^{-1} \otimes \Sigma_t^{-1}) \left[ \frac{(\nu + D)}{\nu - 2 + y_t' \Sigma_t^{-1} y_t} (y_t \otimes y_t) - \text{vec}(\Sigma_t) \right],\end{aligned}\quad (6)$$

where  $\log$  denotes the natural logarithm,  $\Psi_t$  is the derivative  $\partial \text{vech}(\Sigma_t)/\partial f_t'$ ,  $\mathcal{D}_D$  is the duplication matrix as defined in [Abadir and Magnus \(2005\)](#), and  $\otimes$  denotes Kronecker multiplication.

The transition equation (5) adjusts  $f_t$  at every step using the scaled score  $S_t \nabla_t$  of the conditional density at time  $t$ . This can be regarded as a steepest ascent improvement of the parameter using the local (at time  $t$ ) likelihood fit of the model. The scaled score is a martingale difference sequence with mean zero, and acts as an innovation term. The coefficients in  $A$  and  $B$  can be restricted so that the process  $f_t$  is covariance stationary. The initial value  $f_1$  is most conveniently initialized at the unconditional mean of the stationary process.

To accommodate a possibly asymmetric response in the correlation dynamics, we extend (6) slightly as

$$\begin{aligned}\nabla_t &= \frac{1}{2} \Psi_t' \mathcal{D}'_D(\Sigma_t^{-1} \otimes \Sigma_t^{-1}) \left[ \frac{(\nu + k)}{\nu - 2 + y_t' \Sigma_t^{-1} y_t} (y_t \otimes y_t) - \text{vec}(\Sigma_t) \right. \\ &\quad \left. + C \cdot \text{vec}(1(y_t \cdot > 0) \cdot 1(y_t \cdot > 0)') \right],\end{aligned}\quad (7)$$

where  $1(\cdot)$  is an indicator function,  $\cdot >$  implies an element-by-element evaluation, and  $C$  is a scalar (or diagonal matrix) to be estimated. Clearly, (7) reduces to (6) for  $C = 0$ . Values of  $C \neq 0$ , however, allow the correlation dynamics to react differently to increasing versus decreasing marginal risks. If  $C > 0$ , for example, then the correlation parameters increase more quickly in bad times (increasing risks) than in good times (falling risks). Importantly, the asymmetry in the conditional correlations carries over to skewness in the unconditional distribution of  $\tilde{y}_{it}$ , similar to features well-known from the familiar GARCH model with leverage; see e.g. [Glosten et al. \(1993\)](#).

The covariance  $\Sigma_t(f_t)$  is fitted to weekly log-changes in observed bank and sovereign

EDFs. Web Appendix B discusses our univariate modeling strategy for changes in marginal risks. The scaling function  $S_t$  in (5) is available in closed form; see Web Appendix C.

### 3.4 Loss-given-default

Portfolio risk levels depend substantially on the assumptions made in the modeling of the loss (fraction)-given-default. We distinguish two separate cases: bank and sovereign counterparties.

Collateralized lending to banks within the Eurosystem’s liquidity facilities implies a double recourse. If a bank defaults, the central bank can access the pledged collateral and sell it in the market to cover its losses. Conservatively calibrated haircuts on the market value of pledged assets ensure that a sufficient amount of collateral is almost always available to cover losses. Haircuts are higher for more volatile, longer duration, and more credit-risky claims. For example, so-called non-marketable assets carry valuation haircuts of up to 65%. As a result, historical counterparty-level LGDs have been approximately zero for most central banks, owing to conservative ex-ante risk management frameworks and haircuts.

The case of Lehman brothers can serve as an (extreme) example. Its German subsidiary, Lehman Brothers Bankhaus, defaulted on the Eurosystem on 15 September 2008. In the weeks leading up to the default, out-of-fashion mortgage-backed-securities had been posted as collateral. These were highly non-liquid and non-marketable at the time. In addition, an untypically large amount of central bank liquidity had been withdrawn just prior to the default, possibly to help out the U.S. parent. Even so, the posted collateral was ultimately sufficient to recover all losses. The workout-LGD was zero as a result; see [Bundesbank \(2015\)](#).

A substantial loss to the central bank may nevertheless occur in extreme scenarios when both banks and their collateral default simultaneously. This was a valid concern during the sovereign debt crisis. A subset of banks pledged bonds issued by their domestic government, or bank bonds that were eligible because they were also government-guaranteed. This exposed the central bank to substantial “wrong way risk,” as bank and sovereign risks are highly positively dependent in the data.

We incorporate the above observations as follows. For a bank counterparty  $i$ , we model

LGD stochastically as

$$\text{LGD}_{it} = 0.02 + 0.58 \cdot 1 \{ \tilde{y}_{jt} < y_{jt}^* \text{ for at least one SMP country } j \}.$$

i.e.,  $\text{LGD}_{it} = 0.02$  if bank counterparty  $i$  defaults but no sovereign  $j$  defaults. The LGD increases to 60% if bank  $i$  defaults and a (any) euro area sovereign defaults as well (in the same simulation) and a sovereign debt crisis were to ensue as a result. The 2% value for bank workout LGDs is not unrealistically low, as explained above. The 60% stressed LGD is chosen in line with international evidence on sovereign bond haircuts; see e.g. [Cruces and Trebesch \(2013, Table 1\)](#).

In case of a sovereign counterparty, e.g., for government bonds acquired within the SMP, only a single recourse applies. We set the LGD to 60% should such a default be observed,

$$\text{LGD}_{jt} = 0.60 \cdot 1 \{ \tilde{y}_{jt} < y_{jt}^* \text{ for country } j \}.$$

More elaborate specifications for LGD are clearly also possible. The present approach, however, is parsimonious and conservative, while still sufficiently flexible to capture the issue of systematic variation of LGDs with defaults as well as wrong-way risk between banks and sovereigns.

### 3.5 Exposures-at-default

Exposures-at-default  $\text{EAD}_{it}(k)$  in (1) can, but do not have to, coincide with currently observed exposure  $\text{EXP}_{it}(k)$ . Recall that in the case of Lehman Brothers Bankhaus, exposures increased substantially in the weeks prior to the observed default. To keep things simple and interpretable, however, we assume that  $\text{EAD}_{it}(k) = \text{EXP}_{it}(k)$ .

We do not have access to all counterparty-specific exposures (loan amounts) over time in our sample. Instead, we have access to the weekly aggregate exposures at the (country( $j$ ), operation( $k$ )) level. In addition, we know the number of banks  $N_t(j, k)$  that have accessed monetary policy operation  $k$  in week  $t$  and country  $j$ . We therefore proceed under the assumption that exposures  $a_{i,kt}$  for  $i = 1, \dots, N_t(j, k)$  within country  $j$  are Pareto-distributed, in line with e.g. [Janicki and Prescott \(2006\)](#). We thus draw counterparty-specific exposures

according to  $P(a_{i,jkt}) \propto (a_{i,jkt})^{-1/\xi}$  for a given value of  $\xi$  as

$$\text{EXP}_{i,jkt} = \frac{a_{i,jkt}}{\sum_{i=1}^{N_t(j,k)} a_{i,jkt}} \cdot \text{CountryExp}_{jkt}^{\text{observed}},$$

in this way dividing up the observed aggregate bank lending volume per country and policy operation over  $N_t(j, k)$  banks. We chose  $1/\xi = 2$  in line with [Janicki and Prescott \(2006\)](#) to construct the relative shares. We checked that this is approximately consistent with the cross-section of total bank liabilities in the euro area between 2009 and 2015 using the [Hill \(1975\)](#) estimator.

### 3.6 Parameter estimation

Observation-driven multivariate time series models such as (2)–(5) are attractive because the log-likelihood is known in closed form. Parameter estimation is standard as a result. This is a key advantage over alternative parameter-driven risk frameworks, as e.g. considered in [Koopman et al. \(2011\)](#), [Koopman et al. \(2012\)](#), and [Azizpour et al. \(2017\)](#), for which the log-likelihood is not available in closed form and parameter estimation is non-standard. For a given set of observations  $y_1, \dots, y_T$ , the vector of unknown copula parameters  $\theta = \{\omega, A, B, C, \nu\}$  can be estimated by maximizing the log-likelihood function with respect to  $\theta$ , that is

$$\hat{\theta} = \arg \max_{\theta} \sum_{t=1}^T \log p(y_t | f_t; \theta), \quad (8)$$

where  $p(y_t | f_t; \theta) = p(y_t; \Omega(\Sigma_t(f_t)), \nu)$  is the multivariate Student's t density for the vector  $y_t$  containing the observed log-changes in bank and sovereign one-year-ahead EDF measures for all counterparties observed at time  $t$ . The evaluation of  $\log p(y_t | f_t; \theta)$  is easily incorporated in the filtering process for  $f_t$  as described in [Section 3.3](#). The maximization in (8) can be carried out using a conveniently chosen quasi-Newton optimization method. The sub-covariance matrix  $\Omega^{(k)}$  for program  $k$  can be obtained directly from the general matrix  $\Omega(\Sigma_t(f_t))$ .

For the empirical application, we reduce the computational burden of parameter estimation in two ways. First, we proceed in two steps and estimate the parameters of  $D$  marginal models with time varying volatility for each series of log EDF changes. As is standard,

we transform the outcomes using the the probability integral transform and subsequently estimate the copula parameters. [Lucas et al. \(2014\)](#) discuss similar one-step and two-step estimation approaches in a related setting and find that they lead to similar estimates, provided that  $T$  is sufficiently large. Second, we assume that matrices  $A$ ,  $B$ , and  $C$  in the factor transition equation (5) are scalars, such that  $\theta = \{\omega, A, B, C, \nu\} \in \mathbb{R}^5$  is a vector of relatively low dimension.

## 4 Empirical results

Our empirical study is structured around five interrelated questions. What were the expected losses associated with each Eurosystem unconventional monetary policy operation during the sovereign debt crisis? Were the portfolio tail risks at all times covered by Eurosystem financial buffers? How important were spillovers across different monetary policy operations during the sovereign debt crisis? To what extent did unconventional policies differ in terms of ex-post risk efficiency? Finally, do other central banks' policy announcements spill over to the Eurosystem's risks?

### 4.1 Model specification and parameter estimates

For model selection, we are most interested in whether the novel leverage term in (5) is required by the data. Table 1 reports parameter estimates for different specifications of the copula model (2)–(5). The model parameters are estimated from  $9 + 5 = 14$  multivariate time series of daily log changes in banking sector and sovereign EDFs. Univariate Student's  $t$  models with time-varying volatility and leverage are used to model the marginal dynamics for each series separately; see Web Appendix B.

Allowing for an asymmetric response of the correlation factors is strongly preferred by the data based on log-likelihood fit and the reported information criteria of model M2 versus model M1 in Table 1. Web Appendix D shows that model choice can have an economically significant effect on for instance the one-year-ahead expected shortfall estimates. Mean loss estimates are less sensitive. A further generalisation of model M2 to model M3 does not appear to be necessary. The degree-of-freedom parameter  $\nu \approx 20$  in all specifications allows

Table 1: Parameter estimates

Parameter estimates for the copula model (4)–(5) fitted to daily log changes in banking sector and sovereign EDFs. Model M1 enforces a symmetric copula by setting  $C = 0$ . M2 relaxes this constraint by allowing a scalar  $C \neq 0$ . M3 specifies a diagonal matrix  $C$  with three different elements corresponding to sovereign risks, banking sector risks, and their interactions (cross products). Standard errors are constructed from the numerical second derivatives of the log-likelihood function.

	M1	M2	M3
$\omega$	0.985 (0.010)	0.984 (0.011)	0.985 (0.011)
$A$	0.012 (0.001)	0.014 (0.001)	0.014 (0.001)
$B$	1.000 (0.000)	1.000 (0.000)	1.000 (0.000)
$C_1$	-	0.165 (0.056)	0.250 (0.101)
$C_2$	-	-	0.162 (0.066)
$C_3$	-	-	0.243 (0.078)
$\nu$	20.334 (1.763)	20.126 (1.730)	20.201 (1.745)
logLik	1828.45	1833.18	1834.37
AICc	-3648.9	-3656.3	-3654.7
BIC	-3622.5	-3623.3	-3608.5

for a mild degree of joint tail dependence in the copula. Parameter  $B$  is always estimated to be close to 1. Based on the reported likelihood fit and information criteria we select specification M2 for the remainder of the analysis. Using this specification, we combine model parsimony with the ability to explore a rich set of hypotheses given the data at hand.

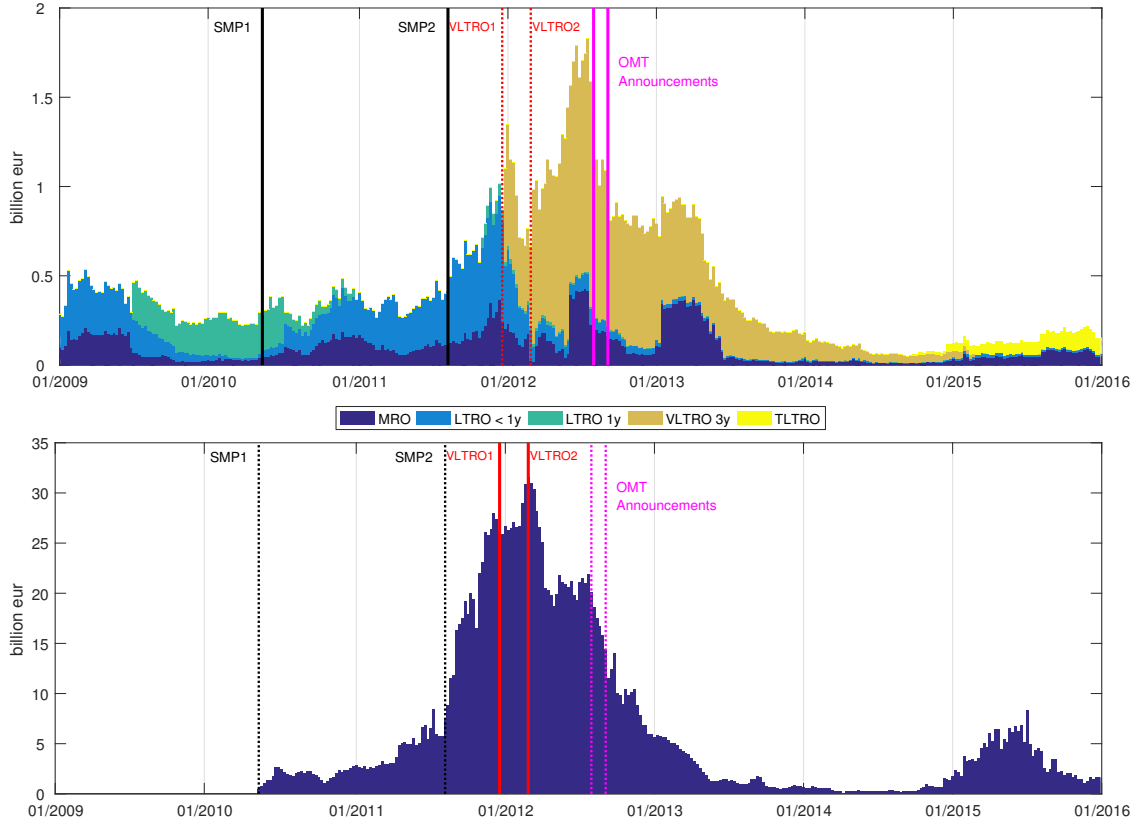
## 4.2 Expected losses

A large and growing literature studies the beneficial impact of ECB unconventional monetary policies during the euro area sovereign debt crisis on financial market and macroeconomic outcomes; see e.g. [Fratzcher and Rieth \(2015\)](#), [Eser and Schwaab \(2016\)](#), [Krishnamurthy et al. \(2018\)](#), among others. By contrast, the potential downsides of unconventional policies, e.g. in terms of increased balance sheet risk, have received less attention.

Figure 4 plots estimated one-year-ahead expected losses from ECB collateralized lending operations (top panel) and SMP asset purchases (bottom panel). Expected losses are additive across operations, and therefore stacked vertically in the top panel of Figure 4. The mean of the loss density is calculated by simulation, using 200,000 draws at each time  $t$ . For each

Figure 4: Expected losses for collateralized lending and SMP portfolio

The top and bottom panels plot the expected losses from liquidity providing operations and the SMP asset holdings, respectively. Vertical lines indicate the announcement of the SMP on 10 May 2010 and its cross-sectional extension on 08 August 2011, the allotment of two VLTROs on 21 December 2011 and 28 February 2012, and two announcements regarding the Outright Monetary Transactions (OMT) in August and September 2012. The vertical axis is in billion euro. Data is weekly between 2009 and 2015.



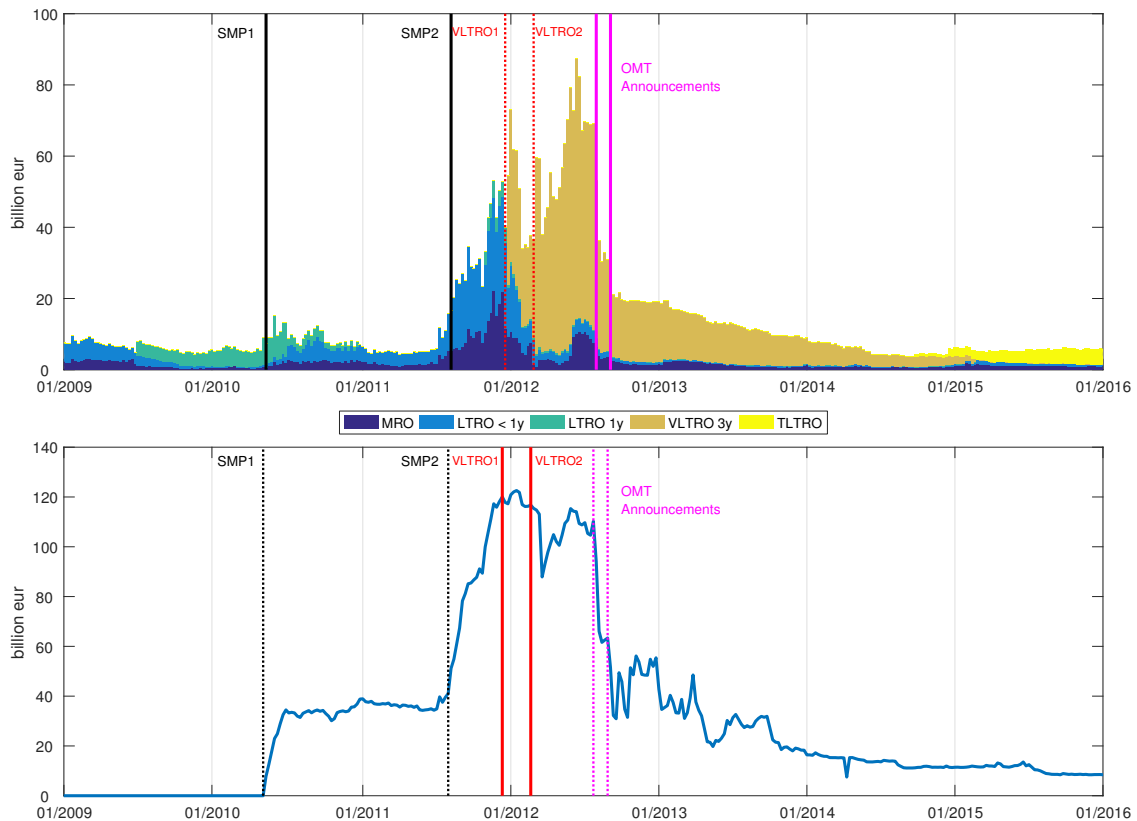
simulation, we keep track of exceedances of  $\tilde{y}_{it}$  below their respective calibrated thresholds at time  $t$  as well as the outcomes for LGD and EAD, as described in Section 3. The risk estimates combine all exposure data, marginal risks, as well as all  $14(14-1)/2=91$  time-varying correlation estimates into a single time series per operation.

The expected losses in Figure 4 reflect, first, a clear deterioration of financial conditions since the beginning of the euro area sovereign debt crisis in the spring of 2010, and second, a clear turning of the tide around mid-2012. Expected losses for both collateralized lending and SMP exposures peak in mid-2012 at around approximately €1.8 bn and €30 bn, respectively. In addition, Figure 4 already hints at the presence of beneficial spillovers across monetary



Figure 5: ES 99% for collateralized lending and SMP portfolio

Top panel: 99% expected shortfall in €bn for five ECB collateralized lending operations. Bottom panel: 99% ES for SMP asset portfolio. Six vertical lines mark the events described in Section 2.1. Data is weekly between 2009 and 2015.



policy operations. For example, the OMT announcements appear to have had a pronounced impact on the expected losses associated with collateralized lending and SMP assets.

### 4.3 Financial buffers and portfolio tail risk

This section studies whether the Eurosystem – the ECB together with its 17 national central banks at the time – was at all times sufficiently able to withstand the materialization of a 99% ES-sized credit loss.

The top and bottom panel of Figure 5 plot the 99% ES associated with five collateralized lending operations and SMP assets. The ES estimates vary widely over time. They peak in 2012 at approximately €80 bn for the collateralized lending operations and approx-

imately €120 bn for SMP assets. All portfolio tail risks collapse sharply following the OMT announcements.

When the ES 99% estimates are scaled by the corresponding policy’s exposure, the SMP assets appear substantially (three times) more tail-risky than the exposures associated with collateralized lending operations. The ratio is even higher (by approximately an order of magnitude) when expected losses are thus scaled and compared. We refer to Web Appendix E for details.

For a commercial bank, financial buffers against a large portfolio loss typically include accounting items such as the current year’s (projected) annual income, revaluation reserves in the balance sheet, general risk provisions, and paid-in equity capital. We adopt a similar notion of financial buffers for the Eurosystem. We recall, however, that a central bank is never liquidity constrained in the currency they issue, so that the notion of financial (solvency) buffers is much less appropriate.

Since the financial crisis in 2008, the Eurosystem as a whole has built up relatively large financial buffers, including from part of the stream of seignorage revenues generated by banknote issuance. Those buffers are mainly in the form of capital and reserves (i.e., paid-up capital, legal reserves and other reserves), revaluation accounts (i.e., unrealized gains on certain assets like gold) and risk provisions. These items stood at €88 bn, €407 bn and €57 bn, respectively, at the end of 2012; see [ECB \(2013b, p. 44\)](#). The overall financial buffers therefore stood at €552 bn. Comparing these balance sheet items with our ES estimates in [Figure 5](#) we conclude that the Eurosystem’s buffers were at all times sufficient to withstand a large (ES99%-sized) credit loss, even at the peak of the euro area sovereign debt crisis.

#### **4.4 Risk spillovers across monetary policy operations**

The riskiness of the ECB’s balance sheet depends on the financial health of its counterparties, which depends in turn on the central bank’s liquidity provision to and asset purchases from those same counterparties. This two-way interdependence can give rise to multiple equilibria as well as a pronounced nonlinearity in the risks inherent in the central bank’s balance sheet. Our time series estimates of portfolio credit risks allow us to identify the impact of ECB policy announcements on the portfolio risk of different monetary policy operations. Table

2 presents one-year-ahead portfolio credit risk estimates (EL and 99% ES) before and after six policy announcements. Web Appendix E presents the analogous results in per cent or corresponding exposures.

We focus on two main results. First, LOLR- and IOLR-implied credit risks are usually negatively related in our sample. Taking risk in one part of the central bank's balance sheet (e.g., the announcement of asset purchases within the SMP) tended to de-risk other positions (e.g., collateralized lending from previous LTROs). Vice versa, the allotment of the two large-scale VLTRO credit operations each decreased the expected shortfall of the SMP asset portfolio. This suggests that central bank risks are convex in exposures. Simply put, doubling size is unlikely to double risk. Similarly, reducing balance sheet size may not reduce risk by as much as one could possibly expect.

Second, a subset of unconventional monetary policies appear to have been self-financing in net risk terms. For example, the first OMT announcement de-risked the Eurosystem's balance sheet by €-32.2 bn (99%-ES). The announcement of OMT technical details on 06 September 2012 was also associated with a strong further reduction of €-21.0 bn in 99% ES. As another example, the allotment of the first VLTRO on 21 December 2011 raised the 99% ES associated with VLTRO lending from zero to approximately €30.6 bn. However, it also sharply reduced the need for shorter-term funding, and de-risked the SMP asset portfolio (as banks invested some of the liquidity in government bonds), such that the overall 99% ES *decreased* by €1.1 bn. We conclude that, in extreme conditions, a central bank can de-risk its balance sheet by doing more.

Such risk reductions are not guaranteed, however. The initial announcement of the SMP on 10 May 2010, for example, and the purchases that were implemented in that week, raised the 99% ES of SMP asset holdings from zero to approximately €7.8 bn. The announcement and initial purchases also de-risked the collateralized lending book, but only mildly so. As a result, the overall 99% ES increased by €5.1 bn.

## 4.5 Risk efficiency

Portfolio risk estimates are a prerequisite for evaluating policy operations in terms of their 'risk efficiency'. Risk efficiency is the principle that a certain amount of expected policy

impact should be achieved with a minimum level of balance sheet risk; see e.g. [ECB \(2015\)](#). Put differently, the impact of any policy operation should be maximal given a certain amount of risk. Given an estimate of policy impact, such as, for example, a change in inflation swap rates or bond yields around the time of a policy announcement, and an estimate of risk, such as, for example, a change in expected losses as reported in [Figure 4](#), different policy operations can be evaluated by scaling the former by the latter. This is approximately similar to the definition of a [Sharpe \(1966\)](#) benefit-to-cost ratio.

Scaling policy impact by additional risk is not unproblematic for at least three reasons. First, ex-ante estimates of impact (and additional risk) are uncertain, particularly if the policy operation is unprecedented. In particular, risks can change after the announcement; see [Section 4.4](#). As a result, ex-ante risk efficiency deliberations may be too uncertain to be of practical use. Second, scaling impact by additional risk may create the impression that both are equally important. This is not the case. Recall that, unlike for commercial banks, risk and profitability are not first-order measures of success for a central bank. Finally, asset purchase programs and credit operations are hard to compare in terms of ex-post risk efficiency. In the case of purchase programs, the impact incorporates the market's expectation about future purchases, while the associated credit risks only accumulate slowly over time. By contrast, the impact and additional risks of novel credit operations are more closely aligned. We study the efficiency of purchase programs and credit operations separately as a result.

[Table 3](#) reports four risk efficiency ratios for the six policy announcements discussed in [Section 2](#). Policy impact could be assessed in many different ways. For example, impact could be proxied by the change in five-year five-year-out inflation swap rates. This reflects the intuition that all monetary policy operations, including unconventional ones, are ultimately tied to the ECB's single mandate of ensuring price stability, and that inflation was below target during the crisis. Alternatively, policy impact could be proxied by how much five-year benchmark bond yields decreased in stressed (GIIPS) countries. This reflects the intuition that each policy operation was implemented during an escalating sovereign debt crisis. Both impacts are reported in [Table 3](#) in percentage points. Additional risk could similarly be assessed in different ways. We consider the expected loss and the 99% expected shortfall. The 'risk efficiency ratios' in the final column of [Table 3](#) are expressed in basis points per €1 bn (for changes in inflation swap rates), and in percentage points per €bn (for changes

in GIIPS bond yields).

We focus on three findings. First, the OMT program dominates the VLTROs and the SMP in terms of ex-post risk efficiency. The ex-post risk efficiency ratios can be negative, and the two OMT-related announcements are a case in point: Both announcements shifted long-term inflation expectations from deflationary tendencies toward the ECB’s target of close to but below two percent (beneficial), decreased five-year sovereign benchmark bond yields of stressed euro area countries (also beneficial), while *removing* risk from the central bank’s balance sheet. By contrast, the SMP, despite its benefits documented elsewhere (e.g. [Eser and Schwaab \(2016\)](#)), does not appear to have been a particularly risk-efficient policy ex post.

Second, the initial announcement of the SMP in 2010 appears to have been far more risk efficient than its later cross-sectional extension in 2011.<sup>4</sup> This is intuitive. The second installment of the SMP focused on deeper and relatively less stressed debt markets. As a result, more bonds had to be purchased to have the same effect on liquidity risk premia.

Finally, the first allotment of VLTRO decreased bond yields somewhat without (initially) increasing the Eurosystem’s expected loss and 99% ES. It therefore appear to be more risk efficient than its second installment.

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<sup>4</sup>Since the aim of the SMP was to add “depth and liquidity” to stressed government bond markets, it is natural to focus on its impact on GIIPS bond yields rather than a change in inflation swap rates.

Table 2: Portfolio credit risks around key policy announcements

Portfolio credit risks for different monetary policy operations around six policy announcements: the SMP announcement on 10 May 2010, the cross-sectional extension of the SMP on 08 August 2011, the allocation of the first VLTRO on 20 December 2011 and of the second VLTRO on 20 February 2012, OMT announcement on 02 August 2012, and the announcement of the OMT's technical details on 06 September 2012.

<b>SMP1</b>	07/05/2010		14/05/2010		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	-	-	0.4 [0.4 0.4]	7.8 [7.7 7.9]	0.4	7.8
MRO	0.0 [0.0 0.0]	0.9 [0.7 1.0]	0.0 [0.0 0.0]	0.7 [0.7 0.9]	0.0	-0.2
LTRO<1y	0.0 [0.0 0.0]	0.6 [0.5 0.8]	0.0 [0.0 0.0]	0.6 [0.5 0.6]	0.0	-0.0
LTRO1y	0.2 [0.2 0.2]	7.6 [6.8 8.3]	0.2 [0.2 0.2]	5.1 [4.8 6.0]	-0.0	-2.5
Total	0.3	9.1	0.7	14.2	0.4	5.1

<b>SMP2</b>	05/08/2011		12/08/2011		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	5.7 [5.7 5.8]	41.1 [40.8 41.5]	7.4 [7.3 7.4]	51.1 [50.6 51.3]	1.6	10.1
MRO	0.1 [0.1 0.1]	5.3 [4.9 5.8]	0.1 [0.1 0.1]	5.2 [4.7 6.1]	-0.0	-0.1
LTRO<1y	0.3 [0.3 0.3]	10.4 [9.0 10.5]	0.3 [0.3 0.3]	11.5 [10.2 12.3]	0.0	1.1
Total	6.1	56.8	7.8	67.9	1.6	11.1

<b>VLTRO1</b>	16/12/2011		30/12/2011		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	27.4 [27.4 27.5]	120.1 [119.6 120.5]	25.9 [25.8 25.9]	117.3 [116.9 117.8]	-1.5	-2.8
MRO	0.4 [0.3 0.4]	22.0 [19.0 23.5]	0.2 [0.2 0.2]	9.3 [9.2 10.2]	-0.2	-12.7
LTRO<1y	0.6 [0.6 0.6]	26.5 [23.7 29.2]	0.4 [0.3 0.4]	13.8 [12.0 14.8]	-0.2	-12.8
LTRO1y	0.1 [0.1 0.1]	4.3 [4.4 5.0]	0.0 [0.0 0.0]	0.8 [0.8 0.9]	-0.0	-3.4
VLTRO3y	-	-	0.5 [0.5 0.6]	31.9 [30.6 36.8]	0.5	30.6
Total	28.4	172.9	27.0	171.8	-1.5	-1.1

Table 2: Portfolio credit risks around key policy announcements; etd.

Portfolio credit risks for different monetary policy operations around six policy announcements: the SMP announcement on 10 May 2010, the cross-sectional extension of the SMP on 08 August 2011, the allocation of the first VLTRO on 20 December 2011 and of the second VLTRO on 20 February 2012, OMT announcement on 02 August 2012, and the announcement of the OMT's technical details on 06 September 2012.

<b>VLTRO2</b>	24/02/2012		09/03/2012		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	30.9	116.7	31.0	114.7	0.1	-2.0
	[30.8 30.9]	[116.0 117.7]	[30.9 31.0]	[113.8 115.5]		
MRO	0.2	7.5	0.0	0.9	-0.2	-6.6
	[0.2 0.2]	[7.0 8.4]	[0.0 0.0]	[0.7 0.9]		
LTRO<1y	0.2	6.2	0.1	3.1	-0.1	-3.1
	[0.2 0.2]	[5.8 6.9]	[0.1 0.1]	[2.8 3.2]		
LTRO1y	0.0	0.6	0.0	0.6	0.0	0.0
	[0.0 0.0]	[0.5 0.6]	[0.0 0.0]	[0.6 0.7]		
VLTRO3y	0.4	23.4	0.9	55.0	0.5	31.6
	[0.4 0.4]	[19.6 25.5]	[0.8 0.9]	[48.7 57.0]		
Total	31.6	154.5	32.0	174.3	0.3	19.8

<b>OMT1</b>	27/07/2012		03/08/2012		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	21.9	110.2	20.1	94.1	-1.8	-16.1
	[21.8 21.9]	[109.5 111.2]	[20.0 20.1]	[92.2 95.1]		
MRO	0.2	7.4	0.2	6.2	0.0	-1.1
	[0.2 0.2]	[6.8 7.9]	[0.2 0.2]	[5.2 6.3]		
LTRO<1y	0.1	3.5	0.1	2.4	-0.0	-1.1
	[0.1 0.1]	[3.2 3.9]	[0.1 0.1]	[2.2 2.8]		
LTRO1y	0.0	0.4	0.0	0.3	-0.0	-0.1
	[0.0 0.0]	[0.3 0.4]	[0.0 0.0]	[0.3 0.3]		
VLTRO3y	1.3	57.9	1.1	44.1	-0.2	-13.8
	[1.2 1.3]	[53.1 61.2]	[1.0 1.1]	[37.5 47.3]		
Total	23.5	179.5	21.5	147.2	-2.0	-32.2

<b>OMT2</b>	31/08/2012		07/09/2012		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	15.8	63.4	14.4	51.6	-1.4	-11.8
	[15.8 15.8]	[61.7 63.9]	[14.4 14.5]	[51.2 52.2]		
MRO	0.2	3.5	0.2	2.4	-0.0	-1.1
	[0.2 0.2]	[3.0 3.6]	[0.2 0.2]	[2.4 2.6]		
LTRO<1y	0.1	1.6	0.1	1.2	-0.0	-0.3
	[0.1 0.1]	[1.3 1.7]	[0.1 0.1]	[1.2 1.3]		
LTRO1y	0.0	0.2	0.0	0.1	-0.0	-0.0
	[0.0 0.0]	[0.2 0.2]	[0.0 0.0]	[0.1 0.1]		
VLTRO3y	0.8	25.7	0.7	17.9	-0.2	-7.8
	[0.8 0.9]	[21.8 28.4]	[0.7 0.7]	[17.9 20.8]		
Total	16.9	94.3	15.3	73.3	-1.6	-21.0

Table 3: Risk efficiency ratios

Economic impact, additional balance sheet risk, and efficiency ratios around six policy announcements. Economic impact is proxied by the change in five-year five-year-out euro area inflation swap rates, and alternatively by the decrease in five-year benchmark bond yields in GIIPS countries. GIIPS is an abbreviation for Greece, Ireland, Italy, Portugal, and Spain. Both entries are in percentage points. Additional balance sheet risk is measured in €bn. Double entries are omitted for clarity below the first panel. Efficiency ratios are in basis points per €bn for changes in inflation swap rates, and in percentage points per €bn for changes in GIIPS bond yields.

Purchase programs	Impact		Add. risk	Efficiency ratio impact/risk	
<b>SMP1</b>	d(infl. swap)	0.01	d(EL)	0.4	3.7
07 May to		0.01	d(ES)	5.1	0.3
14 May 10	-d(5y bond yields)	2.66		0.4	6.7
		2.66		5.1	0.5
<b>SMP2</b>	d(infl. swap)	0.09	d(EL)	1.6	5.3
05 Aug to			d(ES)	11.1	0.8
12 Aug 11	-d(5y bond yields)	1.08			0.7
					0.1
<b>OMT1</b>	d(infl. swap)	0.09	d(EL)	-2.0	-4.6
27 Jul to			d(ES)	-32.2	-0.3
03 Aug 12	-d(5y bond yields)	0.21			-0.1
					0.0
<b>OMT2</b>	d(infl. swap)	0.06	d(EL)	-1.6	-3.9
31 Aug to			d(ES)	-21.0	-0.3
07 Sep 12	-d(5y bond yields)	0.88			-0.5
					0.0

Credit operations	Impact		Add. risk	Efficiency ratio impact/risk	
<b>VLTRO1</b>	d(infl. swap)	0.10	d(EL)	-1.5	-6.8
16 Dec to			d(ES)	-1.1	-9.3
30 Dec 11	-d(5y bond yields)	0.02			0.0
					0.0
<b>VLTRO2</b>	d(infl. swap)	-0.11	d(EL)	0.3	-37.6
24 Feb to			d(ES)	19.8	-0.6
09 Mar 12	-d(5y bond yields)	1.00			3.3
					0.1



## 4.6 International risk spillovers

This section studies to what extent monetary policy announcements of *other* central banks can spill over to affect the Eurosystem’s risks. To this purpose we focus on three important policy announcements: the Federal Reserve’s announcement of “QE3” combined with forward guidance on short term interest rates on 13 September 2012, the “taper tantrum” following Fed communication on 22 May 2013, and the Swiss National Bank’s announcement to stop defending the peg of the Swiss franc to the euro on 15 January 2015. Each of these had major impact on asset prices and volatility at the time, and could in principle have impacted the ECB’s risks.

Table 4 reports our risk estimates around these announcements. Overall, other central banks’ policy announcements appear to have had only a minor impact on Eurosystem risks. Changes in expected losses are typically below €1 bn. At times, the impact is within the reported Monte Carlo uncertainty bands. A potential exception is the Fed’s QE3 and forward guidance announcement. At first glance, this announcement appears to have had a strong beneficial impact on the Eurosystem’s 99% ES. It is equally likely, however, that the risk reduction is still related to the ECB’s second OMT announcement one week earlier.

Table 4: Portfolio credit risks around other central banks' announcements

Portfolio credit risks for different monetary policy operations around three policy announcements: *i*) the Federal Reserve's QE3 and forward guidance announcement on 13 September 2012; *ii*) the Federal Reserve's announcement of their withdrawal from the bond market on 22 May 2013, which resulted in the so called 'taper tantrum'; and *iii*) the announcement by the Swiss National Bank to unpeg the Swiss Franc from the Euro on 15 January 2015.

<b>Fed QE3 &amp; FG, (following OMT2)</b>	07/09/2012		14/09/2012		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	14.4 [14.4 14.5]	51.6 [51.1 52.3]	11.5 [11.5 11.6]	32.3 [31.4 32.4]	-2.9	-19.3
MRO	0.2 [0.2 0.2]	2.4 [2.4 2.6]	0.1 [0.1 0.1]	2.5 [2.4 2.5]	-0.0	0.1
LTRO<1y	0.1 [0.0 0.1]	1.2 [1.2 1.3]	0.0 [0.0 0.0]	0.9 [0.9 1.0]	-0.0	-0.3
LTRO1y	0.0 [0.0 0.0]	0.1 [0.1 0.1]	0.0 [0.0 0.0]	0.1 [0.1 0.1]	-0.0	-0.0
VLTRO3y	0.7 [0.7 0.7]	17.9 [17.8 19.9]	0.6 [0.6 0.6]	17.6 [16.4 17.8]	-0.1	-0.3
<b>Total</b>	<b>15.3</b>	<b>73.3</b>	<b>12.4</b>	<b>53.4</b>	<b>-2.9</b>	<b>-19.9</b>

<b>Fed 'taper tantrum'</b>	17/05/2013		24/05/2013		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	1.5 [1.4 1.5]	19.8 [19.4 20.1]	1.2 [1.2 1.2]	22.2 [21.7 22.7]	-0.3	2.4
MRO	0.2 [0.2 0.2]	1.7 [1.7 1.7]	0.2 [0.2 0.2]	1.8 [1.8 1.8]	0.0	0.1
LTRO<1y	0.0 [0.0 0.0]	0.4 [0.4 0.4]	0.0 [0.0 0.0]	0.4 [0.4 0.4]	0.0	0.0
VLTRO3y	0.3 [0.3 0.3]	10.8 [10.7 11.0]	0.3 [0.3 0.3]	11.4 [11.2 11.6]	0.0	0.5
<b>Total</b>	<b>1.9</b>	<b>32.7</b>	<b>1.8</b>	<b>35.7</b>	<b>-0.2</b>	<b>3.1</b>

<b>SNB peg</b>	09/01/2015		16/01/2015		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	2.3 [2.3 2.3]	11.5 [11.4 11.6]	2.1 [2.1 2.1]	11.4 [11.3 11.5]	-0.2	-0.1
MRO	0.0 [0.0 0.0]	1.0 [1.0 1.0]	0.0 [0.0 0.0]	1.0 [1.0 1.2]	-0.0	0.0
LTRO<1y	0.0 [0.0 0.0]	0.5 [0.5 0.5]	0.0 [0.0 0.0]	0.5 [0.5 0.5]	-0.0	0.0
VLTRO3y	0.0 [0.0 0.0]	2.5 [2.4 2.5]	0.0 [0.0 0.0]	2.2 [2.2 2.3]	-0.0	-0.3
TLTRO	0.1 [0.0 0.1]	2.4 [2.3 2.4]	0.0 [0.0 0.0]	2.4 [2.3 2.4]	-0.0	-0.0
<b>Total</b>	<b>2.4</b>	<b>17.9</b>	<b>2.3</b>	<b>17.5</b>	<b>-0.2</b>	<b>-0.4</b>

## 5 Conclusion

We introduced a tractable non-Gaussian framework to track central bank balance sheet risks at a high (weekly) frequency. We applied our framework to a subset of ECB monetary policy operations during the euro area sovereign debt crisis. Our results suggest that central banks can influence their credit risks, particular when they act as lenders- and investors-of-last-resort. Central bank risks are then concave in exposures: doubling size less than doubles risk.

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**Web Appendix to**

**“Risk endogeneity at the  
lender/investor-of-last-resort”**

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Bernd Schwaab, and Xin Zhang

# Web Appendix A: CDS-implied sovereign EDFs

Unfortunately, firm-value based EDF measures are unavailable for sovereigns. We therefore need to infer physical probabilities of default from observed sovereign CDS spreads. This section provides the details how we obtain CDS-implied-EDFs for euro area sovereigns.

We proceed in four steps. First, we invert the CDS pricing formula of [O’Kane \(2008\)](#) to obtain risk-neutral default intensities (or hazard rates). We do this at each point in time for multiple CDS contracts referencing different maturities. Second, we convert the risk-neutral probabilities into physical ones using the nonlinear mapping fitted by [Heynderickx et al. \(2016\)](#). Third, we fit a [Nelson and Siegel \(1987\)](#) curve to the term structure of physical default hazard rates. Finally, we obtain one-year ahead CDS-implied-EDFs as an integral expression over the  $[0,1]$  year interval.

## Step 1

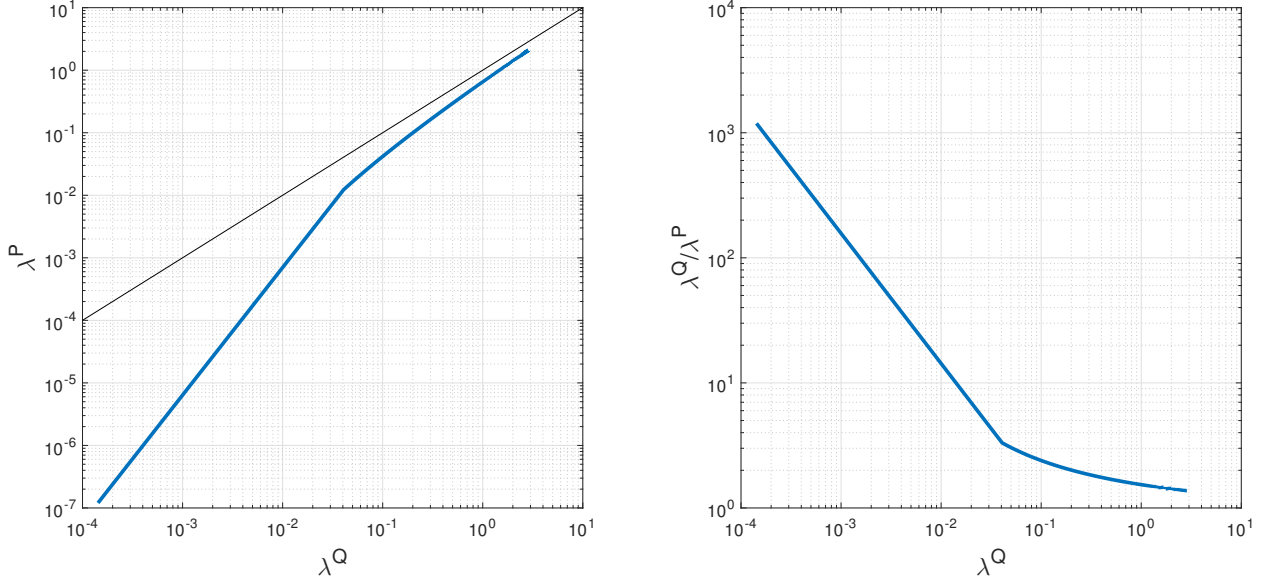
We consider CDS spreads referencing euro area sovereign  $i$  at six different maturities: 1, 2, 3, 4, 5, and 10 years. CDS contracts are denominated in USD and are subject to a full-restructuring credit event clause. CDS spreads are converted into risk-neutral default hazard rates  $\lambda_{it}^Q$  using the procedure described in [O’Kane \(2008\)](#). We need to make choices to implement the procedure in practise. First, we fix a recovery rate at default at a stressed level of 40% for all countries. This is approximately in line with the historical evidence ([Cruces and Trebesch \(2013\)](#)) and also our LGD modeling assumption in Section 3. Second, the term structure of discount rates is assumed to be flat at the one year EURIBOR rate. We use a numerical solver to find the unique default intensity  $\lambda_{it}^Q$  that matches the expected present value of payments within the premium leg to the expected present value of payments within the default leg of the CDS contract.

## Step 2

We convert the risk neutral default intensities  $\lambda_{it}^Q$  into physical default intensities  $\lambda_{it}^P$ . We do so based on the functional form suggested and fitted in [Heynderickx et al. \(2016\)](#). This approach

Figure A.1: Conversion ratio between risk-neutral and physical default hazard rates

Conversion ratio between risk-neutral and physical default hazard rates,  $\lambda^Q$  and  $\lambda^P$  respectively, as extracted from global fit parameters reported in Heynderickx et al. (2016). Left panel:  $\lambda^P$  as a function of  $\lambda^Q$  according to A.1 and A.2. The thin black line corresponds to  $\lambda^P = \lambda^Q$ . Right panel: the ratio  $\frac{\lambda^Q}{\lambda^P}$  is plotted as a function of  $\lambda^Q$ . The ratio increases as the risk neutral intensity decreases.



postulates two non-linear regimes between  $\lambda_{it}^Q$  and  $\lambda_{it}^P$ ,

$$\log\left(\frac{\lambda^Q}{\lambda^P}\right) = a_1 + b_1 \log(\lambda^P) \quad (\text{A.1})$$

$$\log\left(\frac{\lambda^Q}{\lambda^P}\right) = a_2 (\lambda^P)^{b_1} \quad (\text{A.2})$$

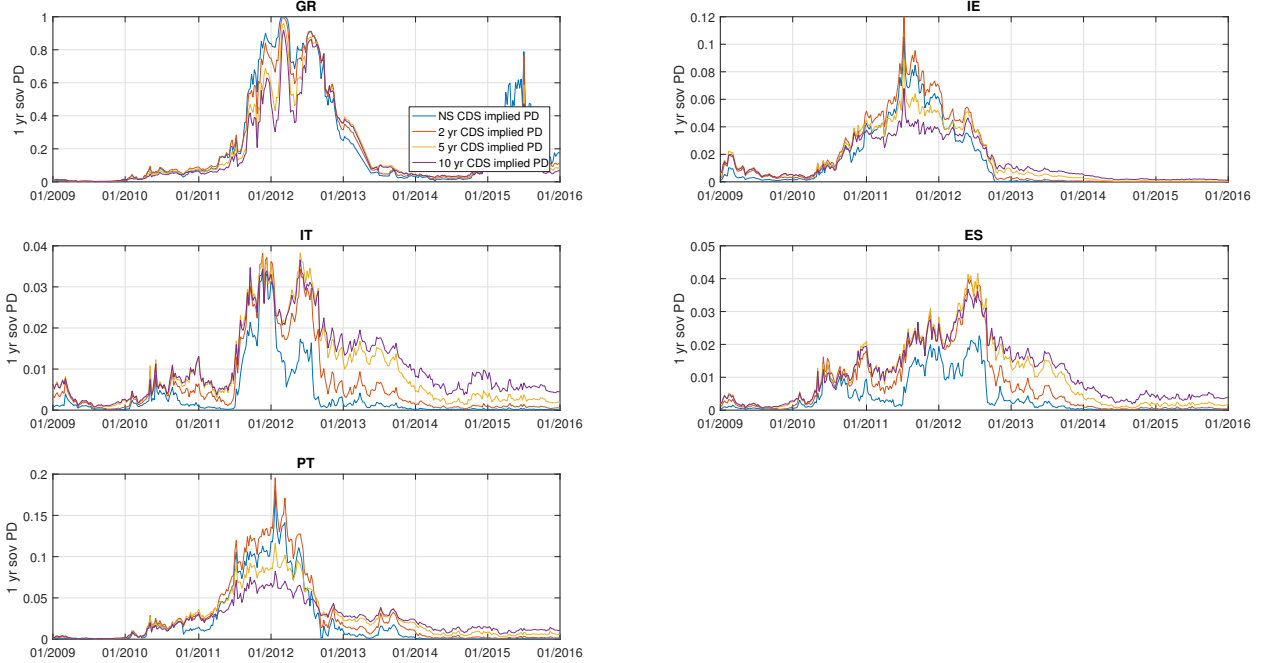
where we have dropped subscripts  $i$  and  $t$  for ease of reading. For small  $\lambda^P$  and  $\lambda^Q$ , A.1 ensures that  $\lambda^P \rightarrow 0$  as  $\lambda^Q \rightarrow 0$ . For large  $\lambda^P$  and  $\lambda^Q$ , A.2 ensures that  $\lambda^P = \lambda^Q$  in the limit of large  $\lambda^Q$ . The value of  $\lambda^Q$  at which the curves cross is a function of parameters  $a_1, b_1, a_2$  and  $b_2$ . We use A.1 for small  $\lambda^Q$  to the left of the crossover point, and A.2 otherwise. The global fit parameters reported in Heynderickx et al. (2016) are  $a_1 = 3.65, b_1 = -0.51, a_2 = 4.16$ , and  $b_2 = -0.26$ .  $\lambda^Q$  and  $\lambda^P$  are measured in inverse-years. Figure A.1 plots the two conversion functions that map risk-neutral to physical default hazard rates.

### Step 3

We fit a Nelson and Siegel (1987) curve to the term structure of physical hazard rates  $\lambda_{\text{NS};it}^P(\tau)$ . This allows us to extract information from CDS spreads referencing different maturities. A curve

Figure A.2: CDS-implied-EDFs for five sovereigns

One-year-ahead CDS-implied-EDFs, plotted in blue. For comparison we show in red, yellow, and purple one-year-ahead EDFs if one were to only use CDS spreads of 2, 5 and 10 year tenors, respectively.



is fitted to each obligor  $i$  at each time  $t$ ,

$$\lambda_{\text{NS}}^P(\tau) = \beta_0 + \beta_1 \tilde{\tau}^{-1} (1 - \exp(-\tilde{\tau})) + \beta_2 [\tilde{\tau}^{-1} (1 - \exp(-\tilde{\tau})) - \exp(-\tilde{\tau})], \quad (\text{A.3})$$

where  $\tilde{\tau} = \tau/T$  and where we dropped the  $i$  and  $t$  subscripts from  $\tau$ . To ensure stable outcomes we impose two additional constraints:  $\lambda_{\text{NS}}^P(\tau)|_{\tau=0} \geq 0$  and  $\frac{\partial \lambda_{\text{NS}}^P(\tau)}{\partial \tau}|_{\tau=0} = 0$ . The first constraint requires the instantaneous hazard rate to be non-negative at all times. The second constraint requires the curve to be upwards-sloping at zero. This restriction rules out negative default hazard rates at the short end of the term structure.

We use a standard solver to obtain parameter estimates for  $\beta_j$  for  $j = 1, 2, 3$  in A.3 over a range of maturities  $T$ . The solver picks the set of  $\{\beta_j, T\}$  that minimizes the least squares difference between  $\lambda_{it}^P$  and  $\lambda_{\text{NS};it}^P$ . We do not model autocorrelation in parameters  $\beta_j$ , but rather fit a new curve for each  $t, T$ , and  $i$ .

## Step 4

Given the fitted curve, we can predict forward-looking physical default probabilities  $\text{PD}_{it}^P(\tau)$  for obligor  $i$  at time  $t$  for any time horizon  $\tau$ . The one-year-ahead CDS-implied-EDF for  $i$  at time  $t$  is given by

$$\text{PD}_i^P(\tau = 1) = 1 - \exp\left(-\int_0^{\tau=1} \lambda_{\text{NS};it}^P(s) ds\right). \quad (\text{A.4})$$

Figure A.2 plots the estimated one-year-ahead EDF for the five SMP countries. In each case, we benchmark our estimate to three alternative EDFs. These alternative EDFs are extracted from only a single CDS contract at the 2, 5, and 10 year maturity, respectively. Naturally, our one-year-ahead EDF corresponds most closely with the 2-year alternative EDF estimate. Interestingly, the term structure of default hazard rates is usually upward sloping. For example, the blue curve is below the other three curves in the Italian or Spanish EDF panels. Occasionally, however, the term structure of default hazard rates is inverted, such as e.g. for Greece during the peak of the crisis. In the latter case, market participants were anticipating the instantaneous probability of default to be decreasing over time.

## Web Appendix B: Univariate modeling

Our univariate marginal modeling strategy is identical to [Lucas et al. \(2017\)](#). We restate it here for convenience. We estimate the parameters of univariate dynamic Student's  $t$  models using log-changes in country  $j$ -specific banking sector EDFs as inputs. For sovereigns, we use log-changes in CDS-implied-EDFs. Parameters are estimated based on maximum likelihood. The univariate models allow us to transform the observations into their probability integral transforms  $\hat{u}_{jt} \in [0, 1]$  based on parameter estimates for  $\sigma_{jt}$  and  $\nu_j$ . The univariate  $t$  density is given by

$$p(y_t; \sigma_t^2, \nu) = \frac{\Gamma((\nu + 1)/2)}{\Gamma(\nu/2)\sqrt{(\nu - 2)\pi\sigma_t^2}} \cdot \left(1 + \frac{y_t^2}{(\nu - 2)\sigma_t^2}\right)^{-\frac{\nu+1}{2}},$$

where  $\sigma_t$  is chosen as  $\sigma_t = \sigma(f_t) = \exp(f_t)$ .

We would like to allow for a leverage (asymmetry) effect also in the univariate score dynamics for volatility. We do so by defining a leverage term that takes nonzero values whenever  $y_t > \tilde{\mu}_t$  with  $\tilde{\mu}_t = 0$ . The score-driven transition equation for this specification are given by

$$\begin{aligned} f_{t+1} &= \tilde{\omega} + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=0}^{q-1} B_j f_{t-j} + C(s_t - s_t^{\tilde{\mu}})1\{y_t > \tilde{\mu}_t\}, \\ s_t &= \mathcal{S}_t \nabla_t, \quad \nabla_t = \partial \log p(y_t | \mathcal{F}_{t-1}; f_t, \theta) / \partial f_t, \\ s_t^{\tilde{\mu}} &= \mathcal{S}_t \nabla_t^{\tilde{\mu}}, \quad \nabla_t^{\tilde{\mu}} = \partial \log p(\tilde{\mu}_t | \mathcal{F}_{t-1}; f_t, \theta) / \partial f_t, \\ \mathcal{S}_t &= \frac{\nu + 3}{2\nu}, \\ \nabla_t &= \frac{(\nu + 1)y_t^2}{(\nu - 2)\sigma_t^2 + y_t^2} - 1. \end{aligned}$$

The above expressions allow for an asymmetric volatility response and skewness in the unconditional data density, see e.g. [Rodriguez and Ruiz \(2012\)](#) for a survey. Univariate filtered volatilities  $\sigma_{jt}$  can be obtained in this way as well.

# Web Appendix C: Score and scaling function for the multivariate t-copula

We estimate the parameters of the multivariate dynamic t-copula model using the pseudo-observations  $\hat{u}_{it} \in [0, 1]$  from on the marginal univariate models as inputs. These pseudo-observations are later transformed to  $t$  distributed random variables  $y_t = F^{-1}(\hat{u}_t; \nu)$ . The  $D$ -dimension multivariate t density is given by

$$p(y_t; \Sigma_t, \nu) = \frac{\Gamma((\nu + D)/2)}{\Gamma(\nu/2)[(\nu - 2)\pi]^{D/2} |\Sigma_t|^{1/2}} \cdot \left[ 1 + \frac{y_t' \Sigma_t^{-1} y_t}{(\nu - 2)} \right]^{-\frac{\nu+D}{2}},$$

where  $\Sigma_t$  is the covariance matrix of  $y_t$  and  $\nu > 2$  is the degree of freedom parameter for the multivariate density. One could rewrite the model in terms of the scaling matrix  $\tilde{\Sigma}_t$  as well. Doing so relaxes the parameter restriction on  $\nu$  to  $\nu > 0$ . Note that the variance is the identity matrix in our copula setting because the univariate models effectively de-volitized the log-changes in bank and sovereign EDFs. As a result, the covariance matrix  $\Sigma_t$  is equivalent to the correlation matrix  $R_t$ .

The score-driven dynamics are incorporated in the covariance matrix in a similar fashion as in [Creal et al. \(2011\)](#), where  $\Sigma_t = \Sigma(f_t)$ . We also include an asymmetry/leverage term in the copula, defined for the adverse case that risks go up, i.e. that  $y_t > 0$ . The dynamic system is given by

$$\begin{aligned} f_{t+1} &= \omega \cdot \text{vech}(\rho(\bar{\Sigma})) + A \cdot S_t \nabla_t + B \cdot f_t, \\ s_t &= S_t \nabla_t, \\ \nabla_t &= \frac{1}{2} \Psi_t' D_D' (\Sigma_t^{-1} \otimes \Sigma_t^{-1}) \left[ \frac{(\nu + k)}{\nu - 2 + y_t' \Sigma_t^{-1} y_t} (y_t \otimes y_t) - \text{vec}(\Sigma_t) \right. \\ &\quad \left. + C \cdot \text{vec}(1(y_t \cdot > 0) \cdot 1(y_t \cdot > 0)') \right], \\ S_t &= \left( \frac{1}{4} \Psi_t' \mathcal{D}_k' (\mathcal{J}_t' \otimes \mathcal{J}_t') [gG - \text{vec}(\mathbf{I}) \text{vec}(\mathbf{I})'] (\mathcal{J}_t \otimes \mathcal{J}_t) \mathcal{D}_k \Psi_t \right)^{-1}, \end{aligned}$$

where  $\Psi_t$  is the derivative  $\partial \text{vech}(\Sigma_t) / \partial f_t'$ ,  $\mathcal{D}_k$  is the duplication matrix and  $\mathcal{J}_t$  is defined as the square root matrix such that  $\Sigma_t^{-1} = \mathcal{J}_t' \mathcal{J}_t$ . The scalar  $g$  is  $(\nu + k) / (\nu + k + 2)$ . The  $k^2 \times k^2$  matrix



$G$  is a particular matrix whose element  $G[\cdot, \cdot]$  is given by

$$G[(i-1) \cdot k + \ell, (j-1) \cdot k + m] = \delta_{ij} \delta_{\ell m} + \delta_{i\ell} \delta_{jm} + \delta_{im} \delta_{j\ell},$$

for  $i, j, \ell, m = 1, \dots, k$ . The Kronecker delta  $\delta_{ij}$  is unity if  $i = j$  or 0 otherwise.

There are a few alternative ways to map  $f_t$  into the covariance matrix  $\Sigma_t$ , and the specific form of  $\Psi_t$  varies accordingly. We adopt a correlation structure based on hyperspherical coordinates  $R_t = X_t' X_t$  where  $X_t(\phi_t)$  is an upper-triangular matrix,

$$X_t = \begin{pmatrix} 1 & c_{12t} & c_{13t} & \cdots & c_{1kt} \\ 0 & s_{12t} & c_{23t}s_{13t} & \cdots & c_{2kt}s_{1kt} \\ 0 & 0 & s_{23t}s_{13t} & \cdots & c_{3kt}s_{2kt}s_{1kt} \\ 0 & 0 & 0 & \cdots & c_{4kt}s_{3kt}s_{2kt}s_{1kt} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & c_{k-1,kt} \prod_{\ell=1}^{k-2} s_{\ell kt} \\ 0 & 0 & 0 & \cdots & \prod_{\ell=1}^{k-1} s_{\ell kt} \end{pmatrix},$$

where  $c_{ijt} = \cos(\phi_{ijt})$  and  $s_{ijt} = \sin(\phi_{ijt})$ . This setup ensures that  $\Sigma_t$  is a proper covariance matrix regardless of  $f_t$ . For example, in the 2-dimensional case, we have the root and correlation matrices given by

$$X_t = \begin{pmatrix} 1 & \cos(\phi_{12,t}) \\ 0 & \sin(\phi_{12,t}) \end{pmatrix}, \quad R_t = X_t' X_t = \begin{pmatrix} 1 & \cos(\phi_{12,t}) \\ \cos(\phi_{12,t}) & 1 \end{pmatrix}, \quad (\text{C.1})$$

with the correlation given by  $\cos(\phi_{12,t})$ .

Under such a parameterization, we can complete the system using the result that

$$\Psi_t = \mathcal{B}_k[(\mathbf{I} \otimes X_t') + (X_t' \otimes \mathbf{I})\mathcal{C}_k]Z_t,$$

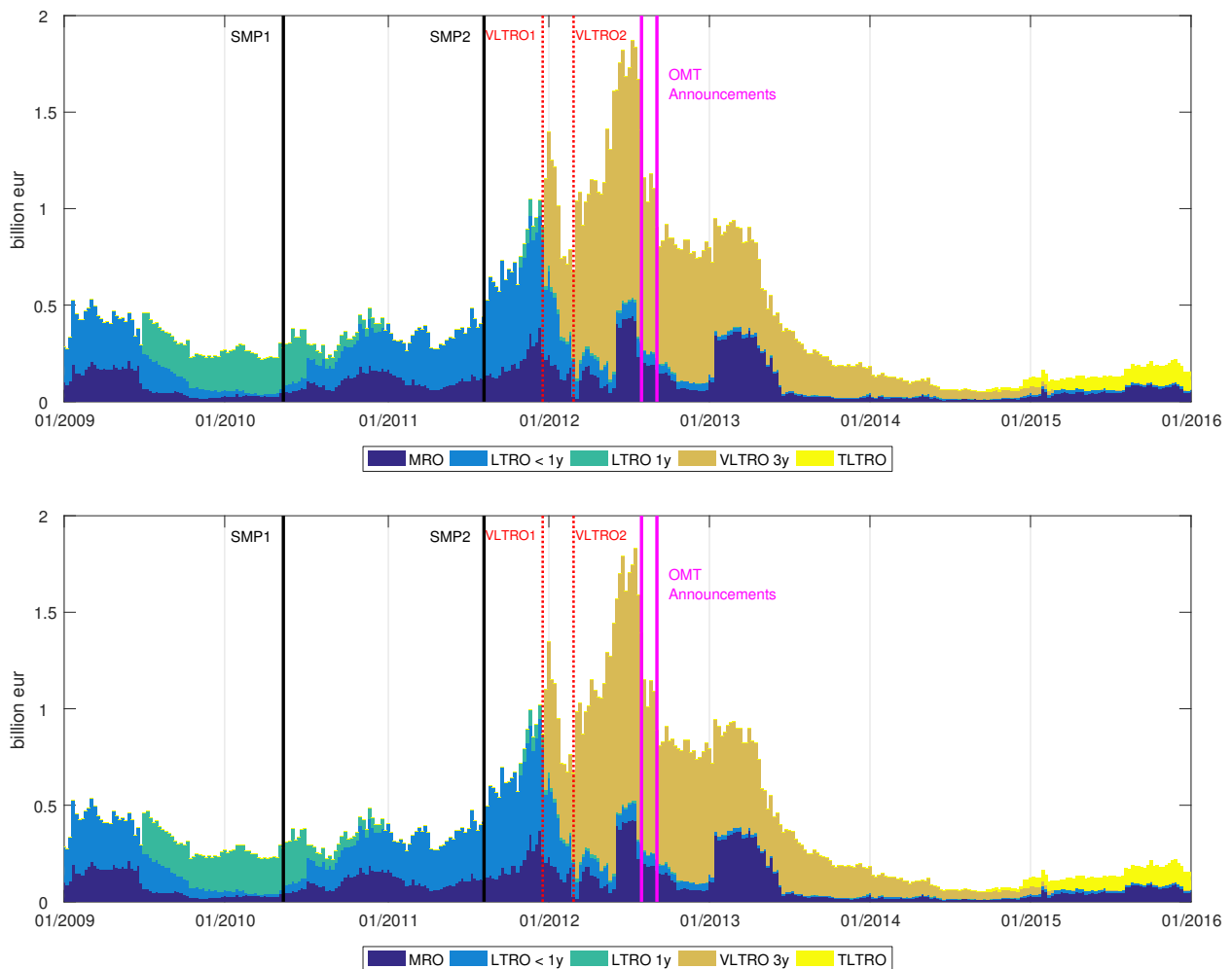
where  $\mathcal{B}_k$  is the elimination matrix,  $\mathcal{C}_k$  is the commutation matrix, and  $Z_t = \partial \text{vec}(X_t) / \partial \phi_t'$ ; see [Abadir and Magnus \(2005\)](#).

# Web Appendix D: Expected losses with and without the leverage term

Figure D.1 provides evidence of the economic significance of the leverage term. The leverage term matters moderately. Instead, portfolio risk is most sensitive to the changes in the marginal probabilities of default  $p_{it}$ , as inferred from the EDFs in levels.

Figure D.1: Expected losses from collateralized lending

The top panel plots the expected losses from liquidity providing operations with the leverage term restricted to zero and re-estimated remaining parameters. The bottom panel plots the expected losses with non-zero leverage terms. Vertical axes are in billion euro. Data is weekly between 2009 and 2015.



# Web Appendix E: Risks in % of size around policy announcements

Figure E.1 reports the ES 99% for collateralized lending (top panel) and SMP assets (bottom panel) expressed in per cent of respective exposures. Table E.1 reports our portfolio credit risk estimates around six key policy announcements. Risks are reported in per cent of exposures.

Figure E.1: ES 99% for collateralized lending and SMP portfolio in per cent of exposures  
 Top panel: 99% ES for five collateralized lending portfolios, each in per cent of respective total exposures.  
 Bottom panel: 99% ES and EL for SMP assets in per cent of nominal (par) value. The vertical lines mark the events described in Section 2.1. Data is weekly between 2009 and 2015.

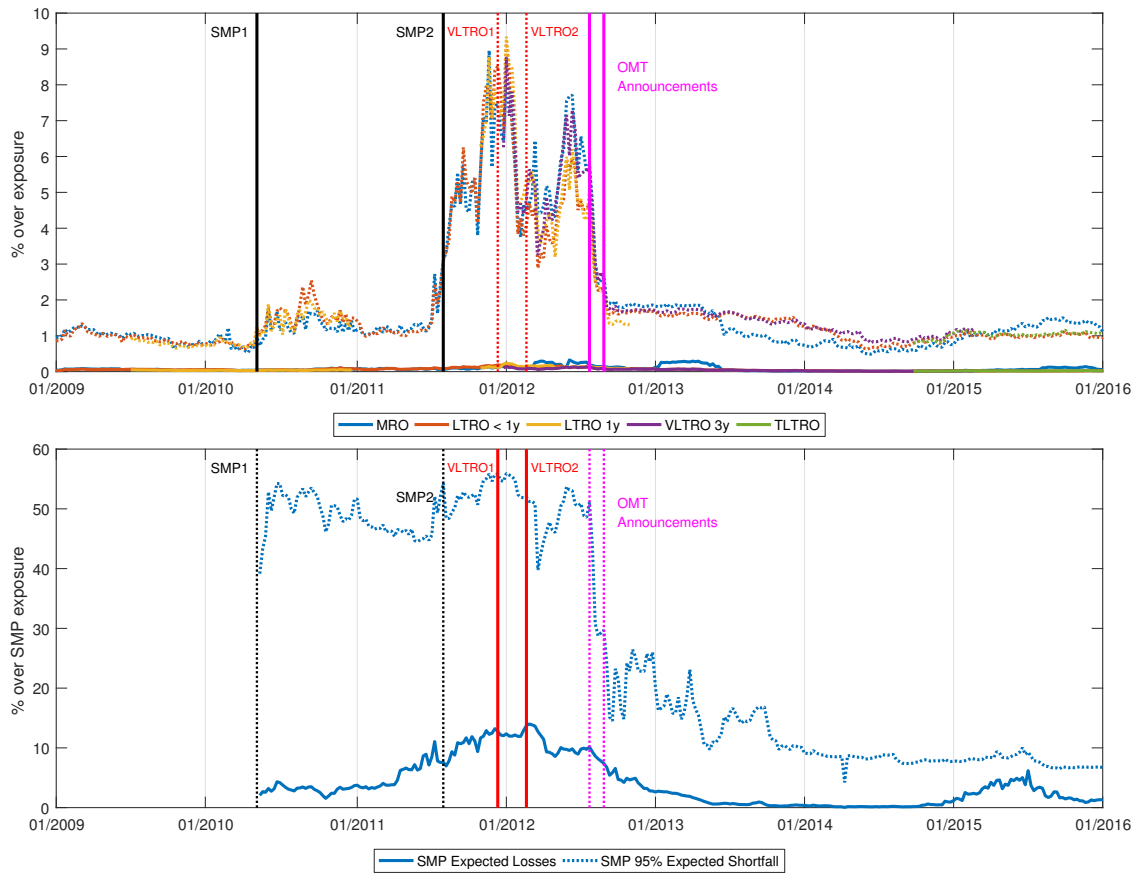


Table E.1: Portfolio credit risks around key policy announcements, in per cent of exposures

Portfolio credit risks for different monetary policy operations around six policy announcements. EL and ES are expressed in per cent of exposures at the time.

<b>SMP1</b>	07/05/2010		14/05/2010		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	-	-	2.0	39.2	2.0	39.2
MRO	0.0	1.0	0.0	0.7	0.0	-0.2
LTRO<1y	0.0	1.2	0.0	1.0	-0.0	-0.1
LTRO1y	0.0	1.2	0.0	1.0	-0.0	-0.2
Total	0.0	1.2	0.1	1.9	0.0	0.7

<b>SMP2</b>	05/08/2011		12/08/2011		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	7.6	54.3	7.0	48.4	-0.6	-5.9
MRO	0.1	3.1	0.1	3.3	0.0	0.2
LTRO<1y	0.1	3.1	0.1	3.4	0.0	0.3
Total	1.1	9.8	1.3	11.2	0.2	1.5

<b>VLTRO1</b>	16/12/2011		30/12/2011		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	12.7	55.8	12.0	54.6	-0.7	-1.2
MRO	0.1	7.5	0.1	6.4	0.0	-1.1
LTRO<1y	0.2	8.5	0.2	6.8	-0.0	-1.8
LTRO1y	0.1	7.5	0.2	7.5	0.1	0.0
Total	3.2	19.7	2.5	16.2	-0.7	-3.6

<b>VLTRO2</b>	24/02/2012		09/03/2012		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	13.7	51.9	13.8	51.2	0.1	-0.7
MRO	0.1	4.5	0.1	5.0	-0.0	0.5
LTRO<1y	0.1	4.1	0.1	4.4	0.0	0.3
LTRO1y	0.2	5.3	0.2	5.6	0.0	0.2
VLTRO3y	0.1	4.8	0.1	5.4	0.0	0.6
Total	3.0	14.8	2.4	13.0	-0.6	-1.8

<b>OMT1</b>	27/07/2012		03/08/2012		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	10.2	51.2	9.3	43.8	-0.8	-7.4
MRO	0.2	5.7	0.2	4.8	0.0	-0.9
LTRO<1y	0.1	4.6	0.1	3.6	-0.0	-1.0
LTRO1y	0.2	5.1	0.1	3.5	-0.0	-1.6
VLTRO3y	0.1	5.8	0.1	4.4	-0.0	-1.4
Total	1.6	12.6	1.5	10.3	-0.1	-2.2

<b>OMT2</b>	31/08/2012		07/09/2012		$\Delta$ EL	$\Delta$ ES(99%)
	EL	ES(99%)	EL	ES(99%)		
SMP	7.4	29.7	6.8	24.2	-0.6	-5.5
MRO	0.1	2.7	0.1	1.9	-0.0	-0.8
LTRO<1y	0.1	2.3	0.1	1.8	-0.0	-0.5
LTRO1y	0.1	2.2	0.1	1.6	-0.0	-0.5
VLTRO3y	0.1	2.6	0.1	1.8	-0.0	-0.8
Total	1.2	6.6	1.1	5.2	-0.1	-1.5